

CHAPTER 5

Analytic Trigonometry

Section 5.1 Using Fundamental Identities

- You should know the fundamental trigonometric identities.

(a) Reciprocal Identities

$$\sin u = \frac{1}{\csc u}$$

$$\csc u = \frac{1}{\sin u}$$

$$\cos u = \frac{1}{\sec u}$$

$$\sec u = \frac{1}{\cos u}$$

$$\tan u = \frac{1}{\cot u} = \frac{\sin u}{\cos u}$$

$$\cot u = \frac{1}{\tan u} = \frac{\cos u}{\sin u}$$

(b) Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

(c) Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$

(d) Negative Angle Identities

$$\sin(-x) = -\sin x$$

$$\csc(-x) = -\csc x$$

$$\cos(-x) = \cos x$$

$$\sec(-x) = \sec x$$

$$\tan(-x) = -\tan x$$

$$\cot(-x) = -\cot x$$

- You should be able to use these fundamental identities to find function values.
- You should be able to convert trigonometric expressions to equivalent forms by using the fundamental identities.
- You should be able to check your answers with a graphing utility.

Vocabulary Check

1. $\sec u$

2. $\tan u$

3. $\cot u$

4. $\csc u$

5. $\tan^2 u$

6. $\csc^2 u$

7. $\sin u$

8. $\sec u$

9. $-\tan u$

10. $\cos u$

$$1. \sin x = \frac{1}{2}, \cos x = \frac{\sqrt{3}}{2}, x \text{ is in Quadrant I.}$$

$$\tan x = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot x = \sqrt{3}$$

$$\csc x = 2$$

$$\sec x = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$2. \csc \theta = 2, \tan \theta = \frac{\sqrt{3}}{3}, \theta \text{ is in Quadrant I.}$$

$$\sin \theta = \frac{1}{2}$$

$$\cot \theta = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\cos \theta = \cot \theta \sin \theta = \frac{\sqrt{3}}{2}$$

$$\sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$3. \sec \theta = \sqrt{2}, \sin \theta = -\frac{\sqrt{2}}{2} \Rightarrow \theta \text{ is in Quadrant IV.}$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$$

$$\cot \theta = \frac{1}{\tan \theta} = -1$$

$$\csc \theta = -\sqrt{2}$$

$$4. \tan x = \frac{\sqrt{3}}{3}, \cos x = -\frac{\sqrt{3}}{2}, x \text{ is in Quadrant III.}$$

$$\sin x = -\sqrt{1 - \left(-\frac{\sqrt{3}}{2}\right)^2} = -\sqrt{\frac{1}{4}} = -\frac{1}{2}$$

$$\csc x = \frac{1}{\sin x} = -2$$

$$\sec x = \frac{1}{\cos x} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\cot x = \frac{1}{\tan x} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$5. \tan x = \frac{7}{24}, \sec x = \frac{-25}{24} \Rightarrow x \text{ is in Quadrant III.}$$

$$\cot x = \frac{24}{7}$$

$$\cos x = -\frac{24}{25}$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\frac{7}{25}$$

$$\csc x = \frac{1}{\sin x} = -\frac{25}{7}$$

$$6. \cot \phi = -5, \sin \phi = \frac{\sqrt{26}}{26}, \phi \text{ is in Quadrant II}$$

$$\cos \phi = \cot \phi \cdot \sin \phi = \frac{-5\sqrt{26}}{26}$$

$$\tan \phi = \frac{1}{\cot \phi} = -\frac{1}{5}$$

$$\csc \phi = \frac{1}{\sin \phi} = \frac{26}{\sqrt{26}} = \sqrt{26}$$

$$\sec \phi = \frac{1}{\cos \phi} = \frac{-26}{5\sqrt{26}} = -\frac{\sqrt{26}}{5}$$

$$7. \sec \phi = -\frac{17}{15}, \sin \phi = \frac{8}{17}, \phi \text{ is in Quadrant II.}$$

$$\cos \phi = -\frac{15}{17}$$

$$\csc \phi = \frac{17}{8}$$

$$\tan \phi = \frac{8/17}{-15/17} = -\frac{8}{15}$$

$$\cot \phi = -\frac{15}{8}$$

8. $\cos\left(\frac{\pi}{2} - x\right) = \frac{3}{5}$, $\cos x = \frac{4}{5}$, x is in Quadrant I.

$$\sin x = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3}{5} \cdot \frac{5}{4} = \frac{3}{4}$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{4}$$

$$\cot x = \frac{1}{\tan x} = \frac{4}{3}$$

9. $\sin(-x) = -\sin x = -\frac{2}{3} \Rightarrow \sin x = \frac{2}{3}$

$$\sin x = \frac{2}{3}, \tan x = -\frac{2\sqrt{5}}{5} \Rightarrow x \text{ is in Quadrant II.}$$

$$\cos x = -\sqrt{1 - \sin^2 x} = -\sqrt{1 - \frac{4}{9}} = -\frac{\sqrt{5}}{3}$$

$$\cot x = \frac{1}{\tan x} = -\frac{\sqrt{5}}{2}$$

$$\sec x = \frac{1}{\cos x} = -\frac{3\sqrt{5}}{5}$$

$$\csc x = \frac{1}{\sin x} = \frac{3}{2}$$

10. $\csc x = 5$, $\cos x > 0$, x is in Quadrant I.

$$\sin x = \frac{1}{\csc x} = \frac{1}{5}$$

$$\cos x = \frac{2\sqrt{6}}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{5} \cdot \frac{5}{2\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\sec x = \frac{1}{\cos x} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\cot x = \frac{1}{\tan x} = 2\sqrt{6}$$

11. $\tan \theta = 2$, $\sin \theta < 0 \Rightarrow \theta$ is in Quadrant III.

$$\sec \theta = -\sqrt{\tan^2 \theta + 1} = -\sqrt{5}$$

$$\cos \theta = \frac{1}{-\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\cot \theta = \frac{1}{2}$$

$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$

$$= -\sqrt{1 - \frac{1}{5}} = -\sqrt{\frac{4}{5}} = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$\csc \theta = -\frac{\sqrt{5}}{2}$$

12. $\sec \theta = -3$, $\tan \theta < 0$, θ is in Quadrant II.

$$\cos \theta = -\frac{1}{3}$$

$$\tan^2 \theta = \sec^2 \theta - 1 = 9 - 1 = 8 \Rightarrow \tan \theta = -\sqrt{8} = -2\sqrt{2}$$

$$\cot \theta = \frac{1}{-2\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

$$\sin \theta = \tan \theta \cos \theta = (-2\sqrt{2})\left(-\frac{1}{3}\right) = \frac{2\sqrt{2}}{3}$$

$$\csc \theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

13. $\csc \theta$ is undefined and $\cos \theta < 0 \Rightarrow \theta = \pi$.

$$\sin \theta = 0$$

$$\cos \theta = -1$$

$$\tan \theta = 0$$

$$\cot \theta \text{ is undefined.}$$

$$\sec \theta = -1$$

- 14.
- $\tan \theta$
- is undefined,
- $\sin \theta > 0$
- .

$$\theta = \frac{\pi}{2}$$

$$\csc \theta = \frac{1}{\sin \theta} = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ is undefined } \Rightarrow \cos \theta = 0, \quad \sec \theta = \frac{1}{\cos \theta} \text{ is undefined.}$$

$$\sin \theta = \sqrt{1 - 0^2} = 1$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{0}{1} = 0$$

15. $\sec x \cos x = \frac{1}{\cos x} \cdot \cos x = 1$

Matches (d).

16. $\tan x \csc x = \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} = \sec x$

Matches (a).

17. $\cot^2 x - \csc^2 x = \cot^2 x - (1 + \cot^2 x) = -1$

Matches (b).

18. $(1 - \cos^2 x) \csc x = \sin^2 x \left(\frac{1}{\sin x} \right) = \sin x$

Matches (f).

19. $\frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$

Matches (e).

20. $\frac{\sin[(\pi/2) - x]}{\cos[(\pi/2) - x]} = \frac{\cos x}{\sin x} = \cot x$

Matches (c).

21. $\sin x \sec x = \sin x \left(\frac{1}{\cos x} \right) = \tan x$

Matches (b).

22. $\cos^2 x (\sec^2 x - 1) = \cos^2 x \tan^2 x = \sin^2 x$

Matches (c).

$$\begin{aligned} 23. \sec^4 x - \tan^4 x &= (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x) \\ &= (\sec^2 x + \tan^2 x)(1) \\ &= \sec^2 x + \tan^2 x \end{aligned}$$

Matches (f).

$$\begin{aligned} 24. \cot x \sec x &= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} \\ &= \csc x \end{aligned}$$

Matches (a).

25. $\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x}{\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} = \sec^2 x$

Matches (e).

$$\begin{aligned} 26. \frac{\cos^2[(\pi/2) - x]}{\cos x} &= \frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \sin x \\ &= \tan x \sin x \end{aligned}$$

Matches (d).

27. $\cot x \sin x = \frac{\cos x}{\sin x} \sin x = \cos x$

28. $\cos \beta \tan \beta = \cos \beta \left(\frac{\sin \beta}{\cos \beta} \right) = \sin \beta$

29. $\sin \phi (\csc \phi - \sin \phi) = \sin \phi \csc \phi - \sin^2 \phi$

$$= \sin \phi \cdot \frac{1}{\sin \phi} - \sin^2 \phi$$

$$= 1 - \sin^2 \phi$$

$$= \cos^2 \phi$$

30. $\sec^2 x (1 - \sin^2 x) = \sec^2 x - \sec^2 x \sin^2 x$

$$= \sec^2 x - \frac{1}{\cos^2 x} \cdot \sin^2 x$$

$$= \sec^2 x - \frac{\sin^2 x}{\cos^2 x}$$

$$= \sec^2 x - \tan^2 x = 1$$

$$31. \frac{\csc x}{\cot x} = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$$

$$32. \frac{\sec \theta}{\csc \theta} = \frac{1}{\cos \theta} \sin \theta = \tan \theta$$

$$\begin{aligned} 33. \sec \alpha \frac{\sin \alpha}{\tan \alpha} &= \frac{1}{\cos \alpha} (\sin \alpha) \cot \alpha \\ &= \frac{1}{\cos \alpha} (\sin \alpha) \left(\frac{\cos \alpha}{\sin \alpha} \right) = 1 \end{aligned}$$

$$\begin{aligned} 34. \frac{\tan^2 \theta}{\sec^2 \theta} &= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sec^2 \theta} \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\frac{1}{\cos^2 \theta}} = \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = \sin^2 \theta \end{aligned}$$

$$35. \sin\left(\frac{\pi}{2} - x\right) \csc x = \cos x \cdot \frac{1}{\sin x} = \cot x$$

$$\begin{aligned} 36. \cot\left(\frac{\pi}{2} - x\right) \cos x &= \tan x \cos x \\ &= \frac{\sin x}{\cos x} \cdot \cos x = \sin x \end{aligned}$$

$$\begin{aligned} 37. \frac{\cos^2 y}{1 - \sin y} &= \frac{1 - \sin^2 y}{1 - \sin y} \\ &= \frac{(1 + \sin y)(1 - \sin y)}{1 - \sin y} \\ &= 1 + \sin y \end{aligned}$$

$$38. \frac{1}{\cot^2 x + 1} = \frac{1}{\csc^2 x} = \sin^2 x$$

$$\begin{aligned} 39. \sin \theta + \cos \theta \cot \theta &= \sin \theta + \cos \theta \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta} \\ &= \frac{1}{\sin \theta} = \csc \theta \end{aligned}$$

$$\begin{aligned} 40. (\sec \theta - \tan \theta)(\csc \theta + 1) &= \frac{1}{\cos \theta} (1 - \sin \theta) \left(\frac{1}{\sin \theta} + 1 \right) \\ &= \frac{1}{\cos \theta} (1 - \sin \theta)(1 + \sin \theta) \frac{1}{\sin \theta} \\ &= \frac{1}{\cos \theta \cdot \sin \theta} (1 - \sin^2 \theta) \\ &= \frac{1}{\cos \theta \sin \theta} \cos^2 \theta = \cot \theta \end{aligned}$$

$$\begin{aligned} 41. \frac{\cos \theta}{1 - \sin \theta} &= \frac{\cos \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} \\ &= \frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta} \\ &= \frac{\cos \theta (1 + \sin \theta)}{\cos^2 \theta} \\ &= \frac{1 + \sin \theta}{\cos \theta} = \sec \theta + \tan \theta \end{aligned}$$

$$\begin{aligned} 42. \frac{1 + \csc \theta}{\cot \theta + \cos \theta} &= \frac{1 + \csc \theta}{\cos \theta (\csc \theta + 1)} \\ &= \frac{1}{\cos \theta} = \sec \theta \end{aligned}$$

$$\begin{aligned}
 43. \quad \frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} &= \frac{1 + 2 \cos \theta + \cos^2 \theta + \sin^2 \theta}{\sin \theta(1 + \cos \theta)} \\
 &= \frac{2 + 2 \cos \theta}{\sin \theta(1 + \cos \theta)} \\
 &= \frac{2(1 + \cos \theta)}{\sin \theta(1 + \cos \theta)} \\
 &= \frac{2}{\sin \theta} = 2 \csc \theta
 \end{aligned}$$

$$\begin{aligned}
 44. \quad \frac{\sin \theta + \cos \theta}{\sin \theta} - \frac{\cos \theta - \sin \theta}{\cos \theta} &= 1 + \cot \theta - 1 + \tan \theta \\
 &= \cot \theta + \tan \theta \\
 &= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{1}{\sin \theta \cos \theta} = \sec \theta \csc \theta
 \end{aligned}$$

$$45. \quad \csc \theta \tan \theta = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

$$46. \quad \sin \theta \csc \theta - \sin^2 \theta = 1 - \sin^2 \theta = \cos^2 \theta$$

$$\begin{aligned}
 47. \quad 1 - \frac{\sin^2 \theta}{1 - \cos \theta} &= \frac{1 - \cos \theta - \sin^2 \theta}{1 - \cos \theta} \\
 &= \frac{\cos^2 \theta - \cos \theta}{1 - \cos \theta} \\
 &= \frac{\cos \theta(\cos \theta - 1)}{1 - \cos \theta} \\
 &= -\cos \theta
 \end{aligned}$$

$$\begin{aligned}
 48. \quad \frac{\tan \theta}{1 + \sec \theta} + \frac{1 + \sec \theta}{\tan \theta} &= \frac{\tan^2 \theta + 1 + 2 \sec \theta + \sec^2 \theta}{(1 + \sec \theta) \tan \theta} \\
 &= \frac{2 \sec^2 \theta + 2 \sec \theta}{(1 + \sec \theta) \tan \theta} \\
 &= \frac{2 \sec \theta(\sec \theta + 1)}{(1 + \sec \theta) \tan \theta} \\
 &= \frac{2 \sec \theta}{\tan \theta} = 2 \csc \theta
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{\cot(-\theta)}{\csc \theta} &= \frac{\cos(-\theta)}{\sin(-\theta)} \sin \theta \\
 &= \frac{\cos \theta}{-\sin \theta} \sin \theta = -\cos \theta
 \end{aligned}$$

$$50. \quad \frac{\csc\left(\frac{\pi}{2} - \theta\right)}{\tan(-\theta)} = \frac{\sec \theta}{-\tan \theta} = -\csc \theta$$

$$51. \cot^2 x - \cot^2 x \cos^2 x = \cot^2 x(1 - \cos^2 x)$$

$$= \frac{\cos^2 x}{\sin^2 x} \sin^2 x = \cos^2 x$$

$$52. \sec^2 x \tan^2 x + \sec^2 x = \sec^2 x(\tan^2 x + 1)$$

$$= \sec^2 x(\sec^2 x) = \sec^4 x$$

$$53. \frac{\cos^2 x - 4}{\cos x - 2} = \frac{(\cos x + 2)(\cos x - 2)}{\cos x - 2} = \cos x + 2$$

$$54. \frac{\csc^2 x - 1}{\csc x - 1} = \frac{(\csc x - 1)(\csc x + 1)}{\csc x - 1} = \csc x + 1$$

$$55. \tan^4 x + 2 \tan^2 x + 1 = (\tan^2 x + 1)^2$$

$$= (\sec^2 x)^2 = \sec^4 x$$

$$56. 1 - 2 \sin^2 x + \sin^4 x = (1 - \sin^2 x)^2$$

$$= (\cos^2 x)^2 = \cos^4 x$$

$$57. \sin^4 x - \cos^4 x = (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$$

$$= (1)(\sin^2 x - \cos^2 x) = \sin^2 x - \cos^2 x$$

$$58. \sec^4 x - \tan^4 x = (\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x) = \sec^2 x + \tan^2 x$$

$$59. \csc^3 x - \csc^2 x - \csc x + 1 = \csc^2 x(\csc x - 1) - (\csc x - 1)$$

$$= (\csc^2 x - 1)(\csc x - 1)$$

$$= \cot^2 x(\csc x - 1)$$

$$60. \sec^3 x - \sec^2 x - \sec x + 1 = \sec^2 x(\sec x - 1) - (\sec x - 1)$$

$$= (\sec^2 x - 1)(\sec x - 1)$$

$$= \tan^2 x(\sec x - 1)$$

$$61. (\sin x + \cos x)^2 = \sin^2 x + 2 \sin x \cos x + \cos^2 x$$

$$= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x$$

$$= 1 + 2 \sin x \cos x$$

$$62. (\tan x + \sec x)(\tan x - \sec x) = \tan^2 x - \sec^2 x$$

$$= -1$$

$$63. (\csc x + 1)(\csc x - 1) = \csc^2 x - 1 = \cot^2 x$$

$$64. (5 - 5 \sin x)(5 + 5 \sin x) = 25 - 25 \sin^2 x$$

$$= 25(1 - \sin^2 x)$$

$$= 25 \cos^2 x$$

$$65. \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = \frac{1 - \cos x + 1 + \cos x}{(1 + \cos x)(1 - \cos x)}$$

$$= \frac{2}{1 - \cos^2 x}$$

$$= \frac{2}{\sin^2 x}$$

$$= 2 \csc^2 x$$

$$66. \frac{1}{\sec x + 1} - \frac{1}{\sec x - 1} = \frac{\sec x - 1 - (\sec x + 1)}{(\sec x + 1)(\sec x - 1)}$$

$$= \frac{\sec x - 1 - \sec x - 1}{\sec^2 x - 1}$$

$$= \frac{-2}{\tan^2 x}$$

$$= -2 \left(\frac{1}{\tan^2 x} \right)$$

$$= -2 \cot^2 x$$

$$67. \tan x - \frac{\sec^2 x}{\tan x} = \frac{\tan^2 x - \sec^2 x}{\tan x} = \frac{-1}{\tan x} = -\cot x$$

$$\begin{aligned} 68. \frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} &= \frac{\cos^2 x + 1 + 2 \sin x + \sin^2 x}{\cos x(1 + \sin x)} \\ &= \frac{2 + 2 \sin x}{\cos x(1 + \sin x)} \\ &= \frac{2}{\cos x} = 2 \sec x \end{aligned}$$

$$\begin{aligned} 69. \frac{\sin^2 y}{1 - \cos y} &= \frac{1 - \cos^2 y}{1 - \cos y} \\ &= \frac{(1 + \cos y)(1 - \cos y)}{1 - \cos y} \\ &= 1 + \cos y \end{aligned}$$

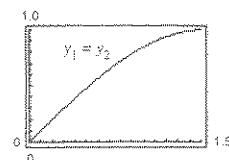
$$\begin{aligned} 70. \frac{5}{\tan x + \sec x} \cdot \frac{\tan x - \sec x}{\tan x - \sec x} &= \frac{5(\tan x - \sec x)}{\tan^2 x - \sec^2 x} \\ &= \frac{5(\tan x - \sec x)}{-1} \\ &= 5(\sec x - \tan x) \end{aligned}$$

$$\begin{aligned} 71. \frac{3}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} &= \frac{3(\sec x + \tan x)}{\sec^2 x - \tan^2 x} \\ &= \frac{3(\sec x + \tan x)}{1} \\ &= 3(\sec x + \tan x) \end{aligned}$$

$$\begin{aligned} 72. \frac{\tan^2 x}{\csc x + 1} \cdot \frac{\csc x - 1}{\csc x - 1} &= \frac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1} \\ &= \frac{\tan^2 x(\csc x - 1)}{\cot^2 x} \\ &= \tan^2 x(\csc x - 1) \tan^2 x \\ &= \tan^4 x(\csc x - 1) \end{aligned}$$

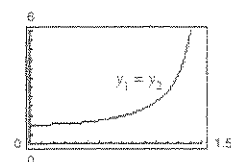
$$73. y_1 = \cos\left(\frac{\pi}{2} - x\right), y_2 = \sin x$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854
y_2	0.1987	0.3894	0.5646	0.7174	0.8415	0.9320	0.9854

Conjecture: $y_1 = y_2$

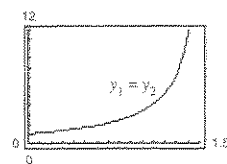
$$74. y_1 = \cos x + \sin x \tan x, y_2 = \sec x$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835
y_2	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835

Conjecture: $y_1 = y_2$

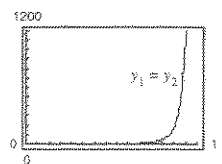
$$75. y_1 = \frac{\cos x}{1 - \sin x}, y_2 = \frac{1 + \sin x}{\cos x}$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814
y_2	1.2230	1.5085	1.8958	2.4650	3.4082	5.3319	11.6814

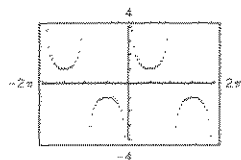
Conjecture: $y_1 = y_2$

76. $y_1 = \sec^4 x - \sec^2 x$, $y_2 = \tan^2 x + \tan^4 x$

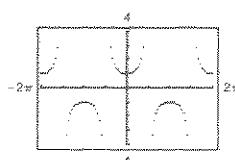
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143
y_2	0.0428	0.2107	0.6871	2.1841	8.3087	50.3869	1163.6143


 Conjecture: $y_1 = y_2$

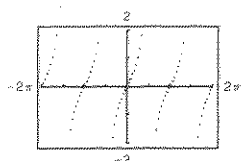
77. $y_1 = \cos x \cot x + \sin x = \csc x$



78. $\sin x(\cot x + \tan x) = \sec x$

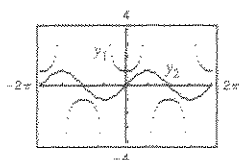


79. $y_1 = \sec x - \frac{\cos x}{1 + \sin x} = \tan x$

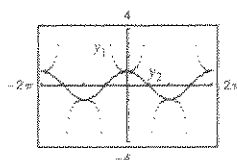


80. $y_1 = \frac{1}{2} \left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \right)$

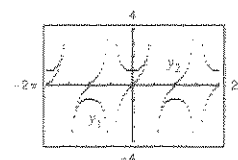
$y_1 \text{ and } y_2 = \sin \theta$



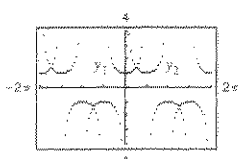
$y_1 \text{ and } y_2 = \cos \theta$



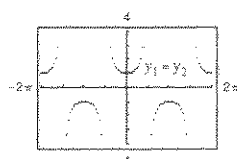
$y_1 \text{ and } y_2 = \tan \theta$



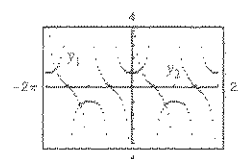
$y_1 \text{ and } y_2 = \frac{1}{\sin \theta} = \csc \theta$



$y_1 \text{ and } y_2 = \frac{1}{\cos \theta} = \sec \theta$



$y_1 \text{ and } y_2 = \frac{1}{\tan \theta} = \cot \theta$



It appears that $\frac{1}{2} \left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \right) = \sec \theta$.

81. $\sqrt{25 - x^2} = \sqrt{25 - (5 \sin \theta)^2}$, $x = 5 \sin \theta$
 $= \sqrt{25 - 25 \sin^2 \theta}$
 $= \sqrt{25(1 - \sin^2 \theta)}$
 $= \sqrt{25 \cos^2 \theta}$
 $= 5 \cos \theta$

82. Let $x = 2 \cos \theta$.

$\sqrt{64 - 16x^2} = \sqrt{64 - 16(4 \cos^2 \theta)}$
 $= 8\sqrt{1 - \cos^2 \theta}$
 $= 8 \sin \theta$

$$\begin{aligned}
 83. \sqrt{x^2 - 9} &= \sqrt{(3 \sec \theta)^2 - 9}, \quad x = 3 \sec \theta \\
 &= \sqrt{9 \sec^2 \theta - 9} \\
 &= \sqrt{9(\sec^2 \theta - 1)} \\
 &= \sqrt{9 \tan^2 \theta} \\
 &= 3 \tan \theta
 \end{aligned}$$

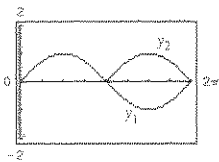
$$\begin{aligned}
 85. \quad x &= 3 \sin \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{9 - x^2} &= \sqrt{9 - 9 \sin^2 \theta} \\
 &= \sqrt{9 \cos^2 \theta} = 3 \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 87. \quad 2x &= 3 \tan \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{4x^2 + 9} &= \sqrt{9 \tan^2 \theta + 9} \\
 &= \sqrt{9 \sec^2 \theta} = 3 \sec \theta
 \end{aligned}$$

$$\begin{aligned}
 89. \quad 4x &= 3 \sec \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{16x^2 - 9} &= \sqrt{9 \sec^2 \theta - 9} \\
 &= \sqrt{9 \tan^2 \theta} = 3 \tan \theta
 \end{aligned}$$

$$\begin{aligned}
 91. \quad x &= \sqrt{2} \sin \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{2 - x^2} &= \sqrt{2 - 2 \sin^2 \theta} \\
 &= \sqrt{2 \cos^2 \theta} = \sqrt{2} \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 93. \quad \sin \theta &= \sqrt{1 - \cos^2 \theta} \\
 \text{Let } y_1 &= \sin x \text{ and } y_2 = \sqrt{1 - \cos^2 x}, \quad 0 \leq x < 2\pi. \\
 y_1 &= y_2 \text{ for } 0 \leq x \leq \pi, \text{ so we have} \\
 \sin \theta &= \sqrt{1 - \cos^2 \theta} \text{ for } 0 \leq \theta \leq \pi.
 \end{aligned}$$



$$\begin{aligned}
 95. \quad \sec \theta &= \sqrt{1 + \tan^2 \theta} \\
 \text{Let } y_1 &= \frac{1}{\cos x} \text{ and } y_2 = \sqrt{1 + \tan^2 x}, \quad 0 \leq x < 2\pi.
 \end{aligned}$$

$$y_1 = y_2 \text{ for } 0 \leq x < \frac{\pi}{2} \text{ and } \frac{3\pi}{2} < x < 2\pi, \text{ so we}$$

$$\text{have } \sec \theta = \sqrt{1 + \tan^2 \theta} \text{ for } 0 \leq \theta < \frac{\pi}{2} \text{ and } \frac{3\pi}{2} < \theta < 2\pi.$$

$$\begin{aligned}
 84. \quad \text{Let } x &= 10 \tan \theta. \\
 \sqrt{x^2 + 100} &= \sqrt{(10 \tan \theta)^2 + 100} \\
 &= \sqrt{100(\tan^2 \theta + 1)} \\
 &= \sqrt{100 \sec^2 \theta} \\
 &= 10 \sec \theta
 \end{aligned}$$

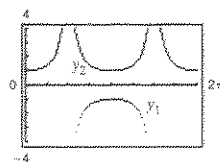
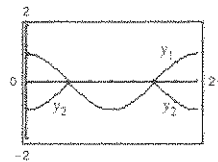
$$\begin{aligned}
 86. \quad x &= 2 \cos \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{4 - x^2} &= \sqrt{4 - 4 \cos^2 \theta} \\
 &= \sqrt{4 \sin^2 \theta} = 2 \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 88. \quad 3x &= 2 \tan \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{9x^2 + 4} &= \sqrt{4 \tan^2 \theta + 4} \\
 &= \sqrt{4 \sec^2 \theta} = 2 \sec \theta
 \end{aligned}$$

$$\begin{aligned}
 90. \quad 3x &= 5 \sec \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{9x^2 - 25} &= \sqrt{25 \sec^2 \theta - 25} \\
 &= \sqrt{25 \tan^2 \theta} = 5 \tan \theta
 \end{aligned}$$

$$\begin{aligned}
 92. \quad x &= \sqrt{5} \cos \theta, \quad 0 < \theta < \frac{\pi}{2} \\
 \sqrt{5 - x^2} &= \sqrt{5 - 5 \cos^2 \theta} \\
 &= \sqrt{5 \sin^2 \theta} = \sqrt{5} \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 94. \quad \cos \theta &= -\sqrt{1 - \sin^2 \theta} \\
 \text{Let } y_1 &= \cos \theta \text{ and } y_2 = -\sqrt{1 - \sin^2 \theta}. \\
 y_1 &= y_2 \text{ for } \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}.
 \end{aligned}$$



$$96. \tan \theta = \sqrt{\sec^2 \theta - 1}$$

$$0 \leq \theta < \frac{\pi}{2}, \pi \leq \theta < \frac{3\pi}{2}$$

$$98. \ln|\csc \theta| + \ln|\tan \theta| = \ln|\csc \theta \cdot \tan \theta| \\ = \ln|\sec \theta|$$

$$100. \ln|\cot t| + \ln(1 + \tan^2 t) = \ln[\cot t(1 + \tan^2 t)] \\ = \ln \frac{(1 + \tan^2 t)}{|\tan t|} \\ = \ln \left| \frac{1}{\tan t} + \frac{\tan^2 t}{\tan t} \right| \\ = \ln|\cot t + \tan t|$$

$$102. \text{ Let } \theta = \frac{2\pi}{3}. \text{ Then}$$

$$\tan \theta = \tan \frac{2\pi}{3} = -\sqrt{3} \neq \sqrt{\sec^2 \theta - 1} = \sqrt{3}.$$

$$104. \text{ Let } \theta = \pi. \text{ Then}$$

$$\sec \theta = \sec \pi = -1 \neq \sqrt{1 + \tan^2 \theta} = 1.$$

$$106. \text{ Let } \theta = \frac{3\pi}{4}. \text{ Then}$$

$$\cot \theta = \cot \frac{3\pi}{4} = -1 \neq \sqrt{\csc^2 \theta - 1} = 1.$$

$$108. \tan^2 \theta + 1 = \sec^2 \theta$$

$$(a) \theta = 346^\circ$$

$$(\tan 346^\circ)^2 + 1 \approx 1.0622$$

$$(\sec 346^\circ)^2 = \left(\frac{1}{\cos 346^\circ} \right)^2 \approx 1.0622$$

$$(b) \theta = 3.1$$

$$(\tan 3.1)^2 + 1 \approx 1.00173$$

$$(\sec 3.1)^2 = \left(\frac{1}{\cos 3.1} \right)^2 \approx 1.00173$$

$$97. \ln|\cos \theta| - \ln|\sin \theta| = \ln \frac{|\cos \theta|}{|\sin \theta|} = \ln|\cot \theta|$$

$$99. \ln(1 + \sin x) - \ln|\sec x| = \ln \left| \frac{1 + \sin x}{\sec x} \right| \\ = \ln|\cos x(1 + \sin x)|$$

$$101. \text{ Let } \theta = \frac{7\pi}{6}. \text{ Then}$$

$$\cos \theta = \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} \neq \sqrt{1 - \sin^2 \theta} = \frac{\sqrt{3}}{2}.$$

$$103. \text{ Let } \theta = \frac{5\pi}{3}. \text{ Then}$$

$$\sin \theta = \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2} \neq \sqrt{1 - \cos^2 \theta} = \frac{\sqrt{3}}{2}.$$

$$105. \text{ Let } \theta = \frac{7\pi}{4}. \text{ Then}$$

$$\csc \theta = \csc \frac{7\pi}{4} = -\sqrt{2} \neq \sqrt{1 + \cot^2 \theta} = \sqrt{2}.$$

$$107. (a) \csc^2 132^\circ - \cot^2 132^\circ \approx 1.8107 - 0.8107 = 1$$

$$(b) \csc^2 \frac{2\pi}{7} - \cot^2 \frac{2\pi}{7} \approx 1.6360 - 0.6360 = 1$$

$$109. \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$(a) \theta = 80^\circ$$

$$\cos(90^\circ - 80^\circ) = \sin 80^\circ$$

$$0.9848 = 0.9848$$

$$(b) \theta = 0.8$$

$$\cos\left(\frac{\pi}{2} - 0.8\right) = \sin 0.8$$

$$0.7174 = 0.7174$$

110. $\sin(-\theta) = -\sin \theta$

(a) $\theta = 250^\circ$

$$\sin(-250^\circ) \approx 0.9397$$

$$-(\sin 250^\circ) \approx 0.9397$$

(b) $\theta = \frac{1}{2}$

$$\sin\left(-\frac{1}{2}\right) \approx -0.4794$$

$$-\left(\sin \frac{1}{2}\right) \approx -0.4794$$

$$\begin{aligned}
 112. \sec x \tan x - \sin x &= \frac{1}{\cos x} \frac{\sin x}{\cos x} - \sin x \\
 &= \sin x [\sec^2 x - 1] \\
 &= \sin x \cdot \tan^2 x
 \end{aligned}$$

114. False

$$\cos 0 \sec \frac{\pi}{4} \neq 1$$

116. As $x \rightarrow 0^+$,

$$\cos x \rightarrow 1 \text{ and } \sec x = \frac{1}{\cos x} \rightarrow 1.$$

118. As $x \rightarrow \pi^+$,

$$\sin x \rightarrow 0 \text{ and } \csc x = \frac{1}{\sin x} \rightarrow -\infty.$$

119. $\sin \theta$

$$\cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \pm \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{\pm 1}{\sqrt{1 - \sin^2 \theta}}$$

$$\cot \theta = \pm \frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$$

The sign + or - depends on the choice of θ .

$$\begin{aligned}
 111. \csc x \cot x - \cos x &= \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} - \cos x \\
 &= \cos x (\csc^2 x - 1) \\
 &= \cos x \cdot \cot^2 x
 \end{aligned}$$

113. True for all $\theta \neq n\pi$

$$\sin \theta \cdot \csc \theta = \sin \theta \left(\frac{1}{\sin \theta} \right) = 1$$

115. As $x \rightarrow \frac{\pi^-}{2}$, $\sin x \rightarrow 1$ and $\csc x \rightarrow 1$.

117. As $x \rightarrow \frac{\pi^-}{2}$, $\tan x \rightarrow \infty$ and $\cot x \rightarrow 0$.

120. $\cos \theta$

$$\sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \pm \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} = \pm \frac{1}{\sqrt{1 - \cos^2 \theta}}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \pm \frac{\cos \theta}{\sqrt{1 - \cos^2 \theta}}$$

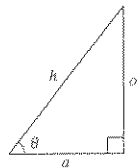
The sign + or - depends on the choice of θ .

$$121. \sin \theta = \frac{\text{opp}}{\text{hyp}}, \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

From the Pythagorean Theorem,

$$(\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$$

$$\sin^2 \theta + \cos^2 \theta = 1.$$



$$122. \sin^2 \theta + \cos^2 \theta = 1$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

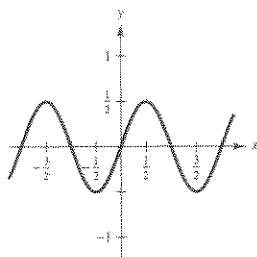
$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$123. f(x) = \frac{1}{2} \sin \pi x$$

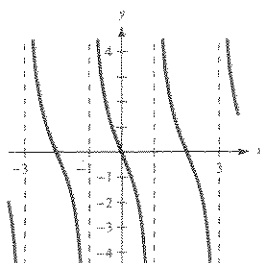
$$\text{Period: } \frac{2\pi}{\pi} = 2$$

$$\text{Amplitude: } \frac{1}{2}$$



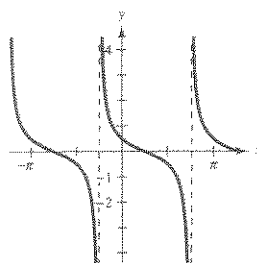
$$124. f(x) = -2 \tan \frac{\pi x}{2}$$

$$\text{Period: } \frac{\pi}{\pi/2} = 2$$



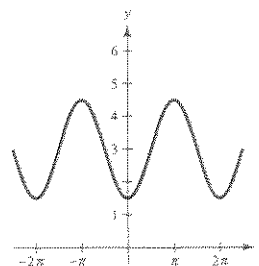
$$125. f(x) = \frac{1}{2} \cot\left(x + \frac{\pi}{4}\right)$$

$$\text{Period: } \pi$$



$$126. f(x) = \frac{3}{2} \cos(x - \pi) + 3$$

$$\text{Amplitude: } \frac{3}{2}$$



Section 5.2 Verifying Trigonometric Identities

- You should know the difference between an expression, a conditional equation, and an identity.
- You should be able to solve trigonometric identities, using the following techniques.
 - Work with *one* side at a time. Do not “cross” the equal sign.
 - Use algebraic techniques such as combining fractions, factoring expressions, rationalizing denominators, and squaring binomials.
 - Use the fundamental identities.
 - Convert all the terms into sines and cosines.

Vocabulary Check

- | | | | |
|----------------|--------------|---------------|-------------|
| 1. conditional | 2. identity | 3. $\cot u$ | 4. $\sin u$ |
| 5. $\tan u$ | 6. $\cos u$ | 7. $\cos^2 u$ | 8. $\cot u$ |
| 9. $-\sin u$ | 10. $\sec u$ | | |

$$1. \sin t \csc t = \sin t \left(\frac{1}{\sin t} \right) = 1$$

$$2. \sec y \cos y = \frac{1}{\cos y} \cos y = 1$$

$$3. \frac{\csc^2 x}{\cot x} = \frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\sin x \cdot \cos x} \\ = \csc x \cdot \sec x$$

$$4. \frac{\sin^2 t}{\tan^2 t} = \frac{\sin^2 t}{\frac{\sin^2 t}{\cos^2 t}} = \cos^2 t$$

$$5. \cos^2 \beta - \sin^2 \beta = (1 - \sin^2 \beta) - \sin^2 \beta \\ = 1 - 2 \sin^2 \beta$$

$$6. \cos^2 \beta - \sin^2 \beta = \cos^2 \beta - (1 - \cos^2 \beta) \\ = 2 \cos^2 \beta - 1$$

$$7. \tan^2 \theta + 6 = (\tan^2 \theta + 1) + 5 \\ = \sec^2 \theta + 5$$

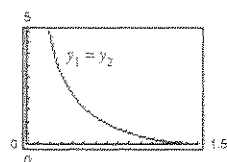
$$8. 2 - \csc^2 z = 2 - (\cot^2 z + 1) = 1 - \cot^2 z$$

$$9. (1 + \sin x)(1 - \sin x) = 1 - \sin^2 x = \cos^2 x$$

$$10. \tan^2 y (\csc^2 y - 1) = \tan^2 y \cot^2 y = 1$$

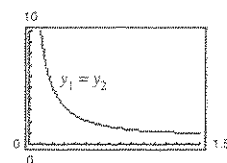
$$11. \frac{1}{\sec x \tan x} = \cos x \cdot \frac{\cos x}{\sin x} \\ = \frac{\cos^2 x}{\sin x} \\ = \frac{1 - \sin^2 x}{\sin x} \\ = \frac{1}{\sin x} - \sin x \\ = \csc x - \sin x$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



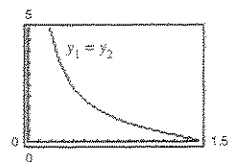
$$12. y_1 = \frac{\csc x - 1}{1 - \sin x} = \frac{\frac{1}{\sin x} - 1}{1 - \sin x} \\ = \frac{1 - \sin x}{\sin x} \cdot \frac{1}{1 - \sin x} \\ = \frac{1}{\sin x} \\ = \csc x = y_2$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148



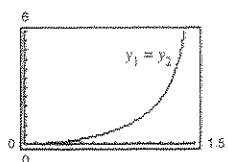
$$\begin{aligned}
 13. \csc x - \sin x &= \frac{1}{\sin x} - \sin x \\
 &= \frac{1 - \sin^2 x}{\sin x} \\
 &= \frac{\cos^2 x}{\sin x} \\
 &= \cos x \cdot \frac{\cos x}{\sin x} \\
 &= \cos x \cdot \cot x
 \end{aligned}$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293



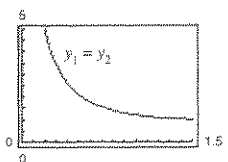
$$\begin{aligned}
 14. y_1 = \sec x - \cos x &= \frac{1}{\cos x} - \cos x \\
 &= \frac{1 - \cos^2 x}{\cos x} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \sin x \left(\frac{\sin x}{\cos x} \right) \\
 &= \sin x \tan x \\
 &= y_2
 \end{aligned}$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135
y_2	0.0403	0.1646	0.3863	0.7386	1.3105	2.3973	5.7135



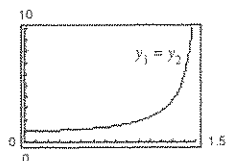
$$\begin{aligned}
 15. \sin x + \cos x \cot x &= \sin x + \cos x \frac{\cos x}{\sin x} \\
 &= \frac{\sin^2 x + \cos^2 x}{\sin x} \\
 &= \frac{1}{\sin x} \\
 &= \csc x
 \end{aligned}$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148
y_2	5.0335	2.5679	1.7710	1.3940	1.1884	1.0729	1.0148

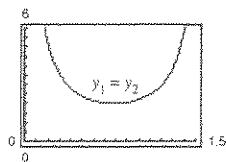


$$\begin{aligned}
 16. y_1 = \cos x + \sin x \tan x \\
 &= \cos x + \frac{\sin^2 x}{\cos x} \\
 &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\
 &= \frac{1}{\cos x} \\
 &= \sec x = y_2
 \end{aligned}$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835
y_2	1.0203	1.0857	1.2116	1.4353	1.8508	2.7597	5.8835

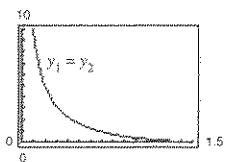


$$\begin{aligned}
 17. \quad \frac{1}{\tan x} + \frac{1}{\cot x} &= \frac{\cot x + \tan x}{\tan x \cdot \cot x} \\
 &= \cot x + \tan x
 \end{aligned}$$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704
y_2	5.1359	2.7880	2.1458	2.0009	2.1995	2.9609	5.9704

$$\begin{aligned}
 18. \quad y_1 &= \frac{1}{\sin x} - \frac{1}{\csc x} \\
 &= \csc x - \sin x = y_2
 \end{aligned}$$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293
y_2	4.8348	2.1785	1.2064	0.6767	0.3469	0.1409	0.0293

19. The error is in line 1: $\cot(-x) \neq \cot x$.

20. There are two errors in line 1:

$$\sec(-\theta) = \sec \theta \text{ and } \sin(-\theta) = -\sin \theta.$$

$$21. \sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \sin^{1/2} x \cos x (1 - \sin^2 x) = \sin^{1/2} x \cos x \cdot \cos^2 x = \cos^3 x \sqrt{\sin x}$$

$$\begin{aligned}
 22. \quad \sec^6 x (\sec x \tan x) - \sec^4 x (\sec x \tan x) &= \sec^4 x (\sec x \tan x) (\sec^2 x - 1) \\
 &= \sec^4 x (\sec x \tan x) \tan^2 x \\
 &= \sec^5 x \tan^3 x
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \cot\left(\frac{\pi}{2} - x\right) \csc x &= \tan x \csc x \\
 &= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \\
 &= \frac{1}{\cos x} = \sec x
 \end{aligned}$$

$$\begin{aligned}
 24. \quad \frac{\sec(\pi/2 - x)}{\tan(\pi/2 - x)} &= \frac{\csc x}{\cot x} \\
 &= \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} \\
 &= \frac{1}{\cos x} = \sec x
 \end{aligned}$$

$$\begin{aligned}
 25. \quad \frac{\csc(-x)}{\sec(-x)} &= \frac{1/\sin(-x)}{1/\cos(-x)} \\
 &= \frac{\cos(-x)}{\sin(-x)} \\
 &= \frac{\cos x}{-\sin x} \\
 &= -\cot x
 \end{aligned}$$

$$\begin{aligned}
 26. \quad (1 + \sin y)[1 + \sin(-y)] &= (1 + \sin y)(1 - \sin y) \\
 &= 1 - \sin^2 y \\
 &= \cos^2 y
 \end{aligned}$$

$$\begin{aligned}
 27. \quad \frac{\cos(-\theta)}{1 + \sin(-\theta)} &= \frac{\cos \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} \\
 &= \frac{\cos \theta(1 + \sin \theta)}{1 - \sin^2 \theta} \\
 &= \frac{\cos \theta(1 + \sin \theta)}{\cos^2 \theta} \\
 &= \frac{1 + \sin \theta}{\cos \theta} \\
 &= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \\
 &= \sec \theta + \tan \theta
 \end{aligned}$$

$$\begin{aligned}
 28. \quad \frac{1 + \csc(-\theta)}{\cos(-\theta) + \cot(-\theta)} &= \frac{1 - \csc \theta}{\cos \theta - \cot \theta} \\
 &= \frac{1 - \csc \theta}{\cos \theta \left(1 - \frac{1}{\sin \theta}\right)} \\
 &= \frac{1 - \csc \theta}{\cos \theta(1 - \csc \theta)} \\
 &= \frac{1}{\cos \theta} \\
 &= \sec \theta
 \end{aligned}$$

$$29. \quad \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$30. \quad \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{1}{\cot x} + \frac{1}{\cot y}}{1 - \frac{1}{\cot x} \cdot \frac{1}{\cot y}} \cdot \frac{\cot x \cot y}{\cot x \cot y} = \frac{\cot y + \cot x}{\cot x \cot y - 1}$$

$$\begin{aligned}
 31. \quad \frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} &= \frac{(\cos x + \cos y)(\cos x - \cos y) + (\sin x + \sin y)(\sin x - \sin y)}{(\sin x + \sin y)(\cos x + \cos y)} \\
 &= \frac{\cos^2 x - \cos^2 y + \sin^2 x - \sin^2 y}{(\sin x + \sin y)(\cos x + \cos y)} \\
 &= \frac{1 - 1}{(\sin x + \sin y)(\cos x + \cos y)} \\
 &= 0
 \end{aligned}$$

$$32. \quad \frac{\tan x + \cot y}{\tan x \cot y} = \frac{1}{\cot y} + \frac{1}{\tan x} = \tan y + \cot x$$

$$\begin{aligned}
 33. \quad \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} &= \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta}} \\
 &= \sqrt{\frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta}} \\
 &= \sqrt{\frac{(1 + \sin \theta)^2}{\cos^2 \theta}} \\
 &= \frac{1 + \sin \theta}{|\cos \theta|}
 \end{aligned}$$

$$\begin{aligned}
 34. \quad \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} &= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta} \cdot \frac{1 - \cos \theta}{1 - \cos \theta}} \\
 &= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} \\
 &= \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}} \\
 &= \frac{1 - \cos \theta}{|\sin \theta|}
 \end{aligned}$$

Note: Check your answer with a graphing utility. What happens if you leave off the absolute value?

$$35. \sin^2\left(\frac{\pi}{2} - x\right) + \sin^2 x = \cos^2 x + \sin^2 x = 1$$

$$36. \sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) = \sec^2 y - \tan^2 y = 1$$

$$37. \sin x \csc\left(\frac{\pi}{2} - x\right) = \sin x \sec x$$

$$38. \sec^2\left(\frac{\pi}{2} - x\right) - 1 = \csc^2 x - 1 = \cot^2 x$$

$$= \sin x \left(\frac{1}{\cos x} \right)$$

$$= \tan x$$

$$39. 2 \sec^2 x - 2 \sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 2 \sec^2 x(1 - \sin^2 x) - (\sin^2 x + \cos^2 x)$$

$$= 2 \sec^2 x(\cos^2 x) - 1$$

$$= 2 \cdot \frac{1}{\cos^2 x} \cdot \cos^2 x - 1$$

$$= 2 - 1 = 1$$

$$40. \csc x(\csc x - \sin x) + \frac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x - \csc x \sin x + 1 - \frac{\cos x}{\sin x} + \cot x$$

$$= \csc^2 x - 1 + 1 - \cot x + \cot x$$

$$= \csc^2 x$$

$$41. \frac{\cot x \tan x}{\sin x} = \frac{1}{\sin x} = \csc x$$

$$42. \frac{1 + \csc \theta}{\sec \theta} - \cot \theta = \cos \theta \left(1 + \frac{1}{\sin \theta} \right) - \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\cos \theta}{\sin \theta} (\sin \theta + 1 - 1)$$

$$= \cos \theta$$

$$43. \csc^4 x - 2 \csc^2 x + 1 = (\csc^2 x - 1)^2$$

$$= (\cot^2 x)^2 = \cot^4 x$$

$$44. \sin x(1 - 2 \cos^2 x + \cos^4 x) = \sin x(1 - \cos^2 x)^2$$

$$= \sin x(\sin^2 x)^2$$

$$= \sin^5 x$$

$$45. \sec^4 \theta - \tan^4 \theta = (\sec^2 \theta + \tan^2 \theta)(\sec^2 \theta - \tan^2 \theta)$$

$$= (1 + \tan^2 \theta + \tan^2 \theta)(1)$$

$$= 1 + 2 \tan^2 \theta$$

$$46. \csc^4 \theta - \cot^4 \theta = (\csc^2 \theta - \cot^2 \theta)(\csc^2 \theta + \cot^2 \theta)$$

$$= \csc^2 \theta + \cot^2 \theta$$

$$= \csc^2 \theta + (\csc^2 \theta - 1)$$

$$= 2 \csc^2 \theta - 1$$

$$47. \frac{\sin \beta}{1 - \cos \beta} \cdot \frac{1 + \cos \beta}{1 + \cos \beta} = \frac{\sin \beta(1 + \cos \beta)}{1 - \cos^2 \beta}$$

$$= \frac{\sin \beta(1 + \cos \beta)}{\sin^2 \beta}$$

$$= \frac{1 + \cos \beta}{\sin \beta}$$

$$48. \frac{\cot \alpha}{\csc \alpha - 1} \cdot \frac{\csc \alpha + 1}{\csc \alpha + 1} = \frac{\cot \alpha(\csc \alpha + 1)}{\csc^2 \alpha - 1}$$

$$= \frac{\cot \alpha(\csc \alpha + 1)}{\cot^2 \alpha}$$

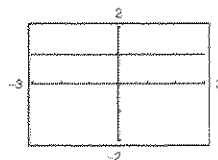
$$= \frac{\csc \alpha + 1}{\cot \alpha}$$

$$49. \frac{\tan^3 \alpha - 1}{\tan \alpha - 1} = \frac{(\tan \alpha - 1)(\tan^2 \alpha + \tan \alpha + 1)}{\tan \alpha - 1} = \tan^2 \alpha + \tan \alpha + 1$$

$$\begin{aligned} 50. \frac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} &= \frac{(\sin \beta + \cos \beta)(\sin^2 \beta - \sin \beta \cos \beta + \cos^2 \beta)}{\sin \beta + \cos \beta} \\ &= \sin^2 \beta + \cos^2 \beta - \sin \beta \cos \beta \\ &= 1 - \sin \beta \cos \beta \end{aligned}$$

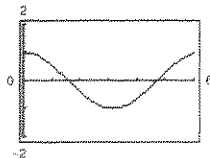
51. It appears that $y_1 = 1$. Analytically,

$$\begin{aligned} \frac{1}{\cot x + 1} + \frac{1}{\tan x + 1} &= \frac{\tan x + 1 + \cot x + 1}{(\cot x + 1)(\tan x + 1)} \\ &= \frac{\tan x + \cot x + 2}{\cot x \tan x + \cot x + \tan x + 1} \\ &= \frac{\tan x + \cot x + 2}{\tan x + \cot x + 2} \\ &= 1. \end{aligned}$$



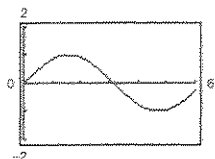
52. The function appears to be $y = \cos x$. Analytically,

$$\begin{aligned} y &= \frac{\cos x}{1 - \tan x} + \frac{\sin x \cdot \cos x}{\sin x - \cos x} \\ &= \frac{\cos x}{1 - (\sin x / \cos x)} + \frac{\sin x \cos x}{\sin x - \cos x} \\ &= \frac{\cos^2 x}{\cos x - \sin x} - \frac{\sin x \cos x}{\cos x - \sin x} \\ &= \frac{\cos x(\cos x - \sin x)}{\cos x - \sin x} = \cos x. \end{aligned}$$



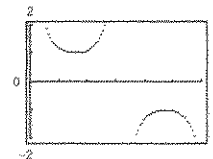
53. It appears that $y_1 = \sin x$. Analytically,

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x} = \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \sin x.$$



54. The function appears to be $y = \csc t$. Analytically,

$$\begin{aligned} y &= \sin t + \frac{\cot^2 t}{\csc t} \\ &= \frac{1 + \cot^2 t}{\csc t} \\ &= \frac{\csc^2 t}{\csc t} = \csc t. \end{aligned}$$



$$\begin{aligned} 55. \ln|\cot \theta| &= \ln\left|\frac{\cos \theta}{\sin \theta}\right| \\ &= \ln\left|\frac{\cos \theta}{\sin \theta}\right| \\ &= \ln|\cos \theta| - \ln|\sin \theta| \end{aligned}$$

$$\begin{aligned} 56. \ln|\sec \theta| &= \ln\left|\frac{1}{\cos \theta}\right| \\ &= \ln|\cos \theta|^{-1} \\ &= -\ln|\cos \theta| \end{aligned}$$

$$57. -\ln(1 + \cos \theta) = \ln(1 + \cos \theta)^{-1}$$

$$= \ln \left[\frac{1}{1 + \cos \theta} \cdot \frac{1 - \cos \theta}{1 - \cos \theta} \right]$$

$$= \ln \frac{1 - \cos \theta}{1 - \cos^2 \theta}$$

$$= \ln \frac{1 - \cos \theta}{\sin^2 \theta}$$

$$= \ln(1 - \cos \theta) - \ln \sin^2 \theta$$

$$= \ln(1 - \cos \theta) - 2 \ln |\sin \theta|$$

$$58. -\ln |\csc \theta + \cot \theta| = -\ln \left| \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right|$$

$$= \ln \left| \frac{1 + \cos \theta}{\sin \theta} \right|^{-1}$$

$$= \ln \left| \frac{\sin \theta}{1 + \cos \theta} \right|$$

$$= \ln \left| \frac{\sin \theta}{1 + \cos \theta} \cdot \frac{1 - \cos \theta}{1 - \cos \theta} \right|$$

$$= \ln \left| \frac{\sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta} \right|$$

$$= \ln \left| \frac{\sin \theta (1 - \cos \theta)}{\sin^2 \theta} \right|$$

$$= \ln \left| \frac{1 - \cos \theta}{\sin \theta} \right|$$

$$= \ln |\csc \theta - \cot \theta|$$

$$59. \sin^2 35^\circ + \sin^2 55^\circ = \cos^2(90^\circ - 35^\circ) + \sin^2 55^\circ \\ = \cos^2 55^\circ + \sin^2 55^\circ = 1$$

$$60. \cos^2 14^\circ + \cos^2 76^\circ = \sin^2(90^\circ - 14^\circ) + \cos^2 76^\circ \\ = \sin^2 76^\circ + \cos^2 76^\circ = 1$$

$$61. \cos^2 20^\circ + \cos^2 52^\circ + \cos^2 38^\circ + \cos^2 70^\circ = \cos^2 20^\circ + \cos^2 52^\circ + \sin^2(90^\circ - 38^\circ) + \sin^2(90^\circ - 70^\circ) \\ = \cos^2 20^\circ + \cos^2 52^\circ + \sin^2 52^\circ + \sin^2 20^\circ \\ = (\cos^2 20^\circ + \sin^2 20^\circ) + (\cos^2 52^\circ + \sin^2 52^\circ) \\ = 1 + 1 = 2$$

$$62. \sin^2 18^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 72^\circ = (\sin^2 18^\circ + \sin^2 72^\circ) + (\sin^2 40^\circ + \sin^2 50^\circ) \\ = (\sin^2 18^\circ + \cos^2 18^\circ) + (\sin^2 40^\circ + \cos^2 40^\circ) \\ = 1 + 1 = 2$$

$$63. \tan^5 x = \tan^3 x \cdot \tan^2 x \\ = \tan^3 x (\sec^2 x - 1) \\ = \tan^3 x \sec^2 x - \tan^3 x$$

$$64. \sec^4 x \tan^2 x = \sec^2 x (1 + \tan^2 x) \tan^2 x \\ = (\tan^2 x + \tan^4 x) \sec^2 x$$

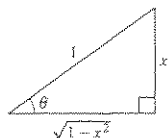
$$65. (\sin^2 x - \sin^4 x) \cos x = \sin^2 x (1 - \sin^2 x) \cos x = \sin^2 x \cdot \cos^2 x \cdot \cos x = \cos^3 x \sin^2 x$$

$$66. 1 - 2 \cos^2 x + 2 \cos^4 x = [1 - 2 \cos^2 x + \cos^4 x] + \cos^4 x = (1 - \cos^2 x)^2 + \cos^4 x = \sin^4 x + \cos^4 x$$

$$67. \text{Let } \theta = \sin^{-1} x \Rightarrow \sin \theta = x = \frac{x}{1}.$$

From the diagram,

$$\tan(\sin^{-1} x) = \tan \theta = \frac{x}{\sqrt{1-x^2}}.$$



$$68. \text{Let } \theta = \sin^{-1} x \Rightarrow \sin \theta = x = \frac{x}{1}.$$

From the diagram,

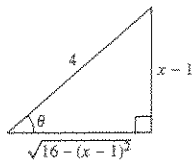
$$\cos(\sin^{-1} x) = \cos \theta = \sqrt{1-x^2}.$$



69. Let $\theta = \sin^{-1} \frac{x-1}{4} \Rightarrow \sin \theta = \frac{x-1}{4}$.

From the diagram,

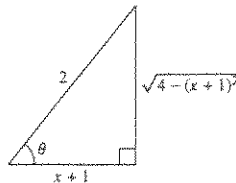
$$\tan\left(\sin^{-1} \frac{x-1}{4}\right) = \tan \theta = \frac{x-1}{\sqrt{16-(x-1)^2}}$$



70. Let $\theta = \cos^{-1} \frac{x+1}{2} \Rightarrow \cos \theta = \frac{x+1}{2}$.

From the diagram,

$$\tan\left(\cos^{-1} \frac{x+1}{2}\right) = \tan \theta = \frac{\sqrt{4-(x+1)^2}}{x+1}$$



71. $\mu W \cos \theta = W \sin \theta$

$$\mu = \frac{W \sin \theta}{W \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta, \quad W \neq 0$$

73. True

75. False. Just because the equation is true for one value of θ , you cannot conclude that the equation is an identity. For example,

$$\sin^2 \frac{\pi}{4} + \cos^2 \frac{\pi}{4} = 1 \neq 1 + \tan^2 \frac{\pi}{4}.$$

$$\begin{aligned} 77. (a) \quad \frac{\sin x}{1 + \cos x} &= \frac{\sin x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} \\ &= \frac{\sin x(1 - \cos x)}{1 - \cos^2 x} \\ &= \frac{\sin x(1 - \cos x)}{\sin^2 x} \\ &= \frac{1 - \cos x}{\sin x} \end{aligned}$$

(b) Not true for $x = 0$ because $(1 - \cos x)/\sin x$ is not defined for $x = 0$.

72. $s = \frac{h \sin(90^\circ - \theta)}{\sin \theta} = h \frac{\cos \theta}{\sin \theta} = h \cot \theta$

74. True. Cosine and secant are even.

76. False. For example, $\sin(1^2) \neq \sin^2(1)$.

$$\begin{aligned} 78. (a) \quad \frac{\tan x}{\sec x - \cos x} &= \frac{\tan x}{\frac{1}{\cos x} - \cos x} \\ &= \frac{\frac{\sin x}{\cos x} \cos x}{1 - \cos^2 x} \\ &= \frac{\sin x}{\sin^2 x} \\ &= \frac{1}{\sin x} \cdot \frac{1/\cos x}{1/\cos x} \\ &= \frac{\sec x}{\tan x} \end{aligned}$$

(b) Not true for $x = \pi$ because $\sec x/\tan x$ is not defined for $x = \pi$.

$$\begin{aligned}
 79. (a) \quad \frac{\sin x}{1 + \cos x} &= \frac{\sin x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} \\
 &= \frac{\sin x(1 - \cos x)}{1 - \cos^2 x} \\
 &= \frac{\sin x(1 - \cos x)}{\sin^2 x} \\
 &= \frac{1 - \cos x}{\sin x} = \csc x - \cot x
 \end{aligned}$$

(b) The identity is true for $x = \frac{\pi}{2}$.

$$\begin{aligned}
 \frac{\sin(\pi/2)}{1 + \cos(\pi/2)} &= \frac{1}{1 + 0} \\
 &= 1 = \csc \frac{\pi}{2} - \cot \frac{\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 81. \quad \sqrt{a^2 - u^2} &= \sqrt{a^2 - a^2 \sin^2 \theta} \\
 &= \sqrt{a^2(1 - \sin^2 \theta)} \\
 &= \sqrt{a^2 \cos^2 \theta} \\
 &= a \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 83. \quad \sqrt{a^2 + u^2} &= \sqrt{a^2 + a^2 \tan^2 \theta} \\
 &= \sqrt{a^2(1 + \tan^2 \theta)} \\
 &= \sqrt{a^2 \sec^2 \theta} \\
 &= a \sec \theta
 \end{aligned}$$

$$85. \quad \sqrt{\tan^2 x} = |\tan x|$$

$$\text{Let } x = \frac{3\pi}{4}. \text{ Then, } \sqrt{\tan^2 x} = \sqrt{(-1)^2} = 1 \neq \tan\left(\frac{3\pi}{4}\right) = -1.$$

$$86. \quad \sin \theta = \sqrt{1 - \cos^2 \theta}.$$

True identity is $\sin \theta = \pm \sqrt{1 - \cos^2 \theta}$.

For example, $\sin \theta \neq \sqrt{1 - \cos^2 \theta}$ for $\theta = \frac{3\pi}{2}$:

$$\sin\left(\frac{3\pi}{2}\right) = -1 \neq \sqrt{1 - 0} = 1$$

$$\begin{aligned}
 88. \quad \sin\left[\frac{(12n+1)\pi}{6}\right] &= \sin\left[\frac{1}{6}(12n\pi + \pi)\right] \\
 &= \sin\left(2n\pi + \frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}
 \end{aligned}$$

$$\text{Thus, } \sin\left[\frac{(12n+1)\pi}{6}\right] = \frac{1}{2} \text{ for all integers } n.$$

$$\begin{aligned}
 80. (a) \quad \frac{\cot x - 1}{\cot x + 1} &= \frac{\cot x - 1}{\cot x + 1} \cdot \frac{\tan x}{\tan x} \\
 &= \frac{1 - \tan x}{1 + \tan x}
 \end{aligned}$$

(b) The identity is true for $x = \frac{\pi}{4}$:

$$\frac{\cot(\pi/4) - 1}{\cot(\pi/4) + 1} = \frac{1 - 1}{1 + 1} = 0$$

$$\frac{1 - \tan(\pi/4)}{1 + \tan(\pi/4)} = \frac{1 - 1}{1 + 1} = 0$$

$$\begin{aligned}
 82. \quad \sqrt{a^2 - u^2} &= \sqrt{a^2 - a^2 \cos^2 \theta} \\
 &= \sqrt{a^2(1 - \cos^2 \theta)} \\
 &= \sqrt{a^2 \sin^2 \theta} \\
 &= a \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 84. \quad \sqrt{u^2 - a^2} &= \sqrt{a^2 \sec^2 \theta - a^2} \\
 &= \sqrt{a^2(\sec^2 \theta - 1)} \\
 &= \sqrt{a^2 \tan^2 \theta} \\
 &= a \tan \theta
 \end{aligned}$$

$$87. \quad \text{When } n \text{ is even, } \cos\left[\frac{(2n+1)\pi}{2}\right] = \cos \frac{\pi}{2} = 0.$$

$$\text{When } n \text{ is odd, } \cos\left[\frac{(2n+1)\pi}{2}\right] = \cos \frac{3\pi}{2} = 0.$$

$$\text{Thus, } \cos\left[\frac{(2n+1)\pi}{2}\right] = 0 \text{ for all } n.$$

$$\begin{aligned}
 89. \quad (x-1)(x-8i)(x+8i) &= (x-1)(x^2 + 64) \\
 &= x^3 - x^2 + 64x - 64
 \end{aligned}$$

Answers will vary.

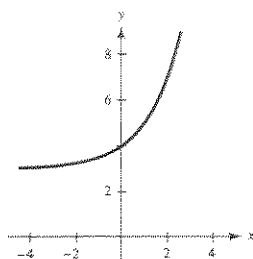
$$90. (x - i)(x + i)(x - 4i)(x + 4i) = (x^2 + 1)(x^2 + 16) = x^4 + 17x^2 + 16$$

$$\begin{aligned} 91. (x - 4)(x - 6 - i)(x - 6 + i) &= (x - 4)((x - 6)^2 + 1) \\ &= (x - 4)(x^2 - 12x + 37) \\ &= x^3 - 16x^2 + 85x - 148 \end{aligned}$$

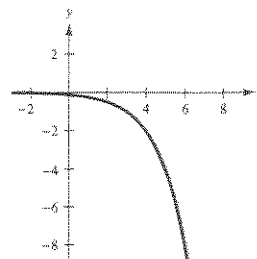
Answers will vary.

$$\begin{aligned} 92. x^2(x - 2)(x - (1 - i))(x - (1 + i)) &= (x^3 - 2x^2)((x - 1)^2 + 1) \\ &= (x^3 - 2x^2)(x^2 - 2x + 2) \\ &= x^5 - 4x^4 + 6x^3 - 4x^2 \end{aligned}$$

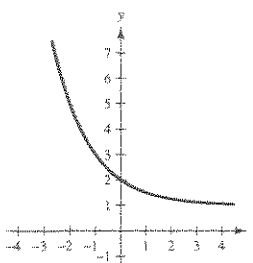
$$93. f(x) = 2^x + 3$$



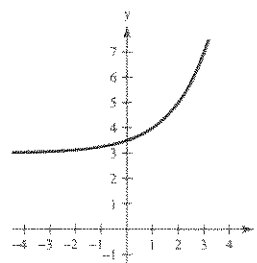
$$94. f(x) = -2^{x-3}$$



$$95. f(x) = 2^{-x} + 1$$



$$96. f(x) = 2^{x-1} + 3$$



$$97. \csc \theta > 0 \text{ and } \tan \theta < 0 \Rightarrow \text{Quadrant II}$$

$$98. \text{Quadrant III}$$

$$99. \sec \theta > 0 \text{ and } \sin \theta < 0 \Rightarrow \text{Quadrant IV}$$

$$100. \text{Quadrant III}$$

Section 5.3 Solving Trigonometric Equations

- You should be able to identify and solve trigonometric equations.
- A trigonometric equation is a conditional equation. It is true for a specific set of values.
- To solve trigonometric equations, use algebraic techniques such as collecting like terms, taking square roots, factoring, squaring, converting to quadratic form, using formulas, and using inverse functions. Study the examples in this section.
- Use your graphing utility to calculate solutions and verify results.

Vocabulary Check

1. general

2. quadratic

3. extraneous

1. $2 \cos x - 1 = 0$

(a) $x = \frac{\pi}{3}: 2 \cos \frac{\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

(b) $x = \frac{5\pi}{3}: 2 \cos \frac{5\pi}{3} - 1 = 2\left(\frac{1}{2}\right) - 1 = 0$

2. $\sec x - 2 = 0$

(a) $x = \frac{\pi}{3}: \sec\left(\frac{\pi}{3}\right) - 2 = \frac{1}{\cos(\pi/3)} - 2$
 $= 2 - 2 = 0$

(b) $x = \frac{5\pi}{3}: \sec\left(\frac{5\pi}{3}\right) - 2 = 2 - 2 = 0$

3. $3 \tan^2 2x - 1 = 0$

(a) $x = \frac{\pi}{12}: 3\left[\tan\left(\frac{2\pi}{12}\right)\right]^2 - 1 = 3 \tan^2 \frac{\pi}{6} - 1 = 3\left(\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$

(b) $x = \frac{5\pi}{12}: 3\left[\tan\left(\frac{10\pi}{12}\right)\right]^2 - 1 = 3 \tan^2 \frac{5\pi}{6} - 1 = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 0$

4. $4 \cos^2 2x - 2 = 0$

(a) $x = \frac{\pi}{8}: 4 \cos^2\left(2 \cdot \frac{\pi}{8}\right) - 2 = 4 \cos^2\left(\frac{\pi}{4}\right) - 2 = 4\left(\frac{\sqrt{2}}{2}\right)^2 - 2 = 0$

(b) $x = \frac{7\pi}{8}: 4 \cos^2\left(2 \cdot \frac{7\pi}{8}\right) - 2 = 4 \cos^2\left(\frac{7\pi}{4}\right) - 2 = 4\left(\frac{\sqrt{2}}{2}\right)^2 - 2 = 0$

5. $2 \sin^2 x - \sin x - 1 = 0$

(a) $x = \frac{\pi}{2}: 2 \sin^2\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right) - 1 = 2 - 1 - 1 = 0$

(b) $x = \frac{7\pi}{6}: 2 \sin^2\left(\frac{7\pi}{6}\right) - \sin\left(\frac{7\pi}{6}\right) - 1 = 2\left(\frac{1}{4}\right) - \left(-\frac{1}{2}\right) - 1 = 0$

6. $\sec^4 x - 3 \sec^2 x - 4 = 0$

(a) $x = \frac{2\pi}{3}: \sec \frac{2\pi}{3} = -2$ and $\sec^4 x - 3 \sec^2 x - 4 = (-2)^4 - 3(-2)^2 - 4 = 0$

(b) $x = \frac{5\pi}{3}: \sec \frac{5\pi}{3} = 2$ and $\sec^4 x - 3 \sec^2 x - 4 = 2^4 - 3(2)^2 - 4 = 0$

7. $\sin x = \frac{1}{2}$

$x = 30^\circ, 150^\circ$

8. $\cos x = \frac{\sqrt{3}}{2}$

$x = 30^\circ, 330^\circ$

9. $\cos x = -\frac{1}{2}$

$x = 120^\circ, 240^\circ$

10. $\sin x = -\frac{\sqrt{2}}{2}$

$x = 225^\circ, 315^\circ$

11. $\tan x = 1$

$x = 45^\circ, 225^\circ$

12. $\tan x = -\sqrt{3}$

$x = 120^\circ, 300^\circ$

13. $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}, \frac{7\pi}{6}$

14. $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

15. $\cot x = -1$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

16. $\sin x = \frac{\sqrt{3}}{2}$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

17. $\tan x = -\frac{\sqrt{3}}{3}$

$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

18. $\cos x = \frac{\sqrt{2}}{2}$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

19. $\csc x = -2 \Rightarrow \sin x = -\frac{1}{2}$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

20. $\sec x = \sqrt{2} \Rightarrow \cos x = \frac{\sqrt{2}}{2}$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

21. $\cot x = \sqrt{3} \Rightarrow \tan x = \frac{\sqrt{3}}{3}$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

22. $\sec x = 2 \Rightarrow \cos x = \frac{1}{2}$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

23. $\tan x = -1$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

24. $\csc x = -\sqrt{2} \Rightarrow \sin x = -\frac{\sqrt{2}}{2}$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

25. $2 \cos x + 1 = 0$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3} + 2n\pi$$

$$\text{or } x = \frac{4\pi}{3} + 2n\pi$$

26. $\sqrt{2} \sin x + 1 = 0$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$x = \frac{5\pi}{4} + 2n\pi$$

$$x = \frac{7\pi}{4} + 2n\pi$$

27. $\sqrt{3} \sec x - 2 = 0$

$$\sqrt{3} \sec x = 2$$

$$\sec x = \frac{2}{\sqrt{3}}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6} + 2n\pi$$

$$\text{or } x = \frac{11\pi}{6} + 2n\pi$$

28. $\cot x + 1 = 0$

$$\cot x = -1$$

$$x = \frac{3\pi}{4} + n\pi$$

29. $3 \csc^2 x - 4 = 0$

$$\csc^2 x = \frac{4}{3}$$

$$\csc x = \pm \frac{2}{\sqrt{3}}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + n\pi \text{ or } x = \frac{2\pi}{3} + n\pi$$

30. $3 \cot^2 x - 1 = 0$

$$\cot^2 x = \frac{1}{3}$$

$$\cot x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{3} + n\pi$$

$$x = \frac{2\pi}{3} + n\pi$$

31. $4 \cos^2 x - 1 = 0$

$$\cos^2 x = \frac{1}{4}$$

$$\cos x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{3} + n\pi \text{ or } x = \frac{2\pi}{3} + n\pi$$

32. $\cos x(\cos x - 1) = 0$

$$\cos x = 0 \quad \text{or} \quad \cos x = 1$$

$$x = \frac{\pi}{2} + n\pi \quad \text{or} \quad x = 2n\pi$$

33. $\sin^2 x = 3 \cos^2 x$

$$\sin^2 x - 3(1 - \sin^2 x) = 0$$

$$4 \sin^2 x = 3$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + n\pi$$

$$\text{or } x = \frac{2\pi}{3} + n\pi$$

34. $(3 \tan^2 x - 1)(\tan^2 x - 3) = 0$

$$\tan^2 x = \frac{1}{3} \quad \text{or} \quad \tan^2 x = 3$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \quad \text{or} \quad \tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{6} + n\pi \quad x = \frac{\pi}{3} + n\pi$$

$$x = \frac{5\pi}{6} + n\pi \quad x = \frac{2\pi}{3} + n\pi$$

35. $\tan x + \sqrt{3} = 0$

$$\tan x = -\sqrt{3}$$

$$x = \frac{2\pi}{3}, \frac{5\pi}{3}$$

36. $2 \sin x + 1 = 0$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

37. $\csc^2 x - 2 = 0$

$$\csc^2 x = 2$$

$$\csc x = \pm \sqrt{2}$$

$$\sin x = \pm \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

38. $\tan^2 x - 1 = 0$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

39. $3 \tan^3 x - \tan x = 0$

$$\tan x(3 \tan^2 x - 1) = 0$$

$$\tan x = 0 \quad \text{or} \quad 3 \tan^2 x - 1 = 0$$

$$x = 0, \pi \quad \tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

40. $2 \sin^2 x = 2 + \cos x$

$$2 - 2 \cos^2 x = 2 + \cos x$$

$$2 \cos^2 x + \cos x = 0$$

$$\cos x(2 \cos x + 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad 2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

41. $\sec^2 x - \sec x - 2 = 0$

$$(\sec x - 2)(\sec x + 1) = 0$$

$$\sec x - 2 = 0 \quad \text{or} \quad \sec x + 1 = 0$$

$$\sec x = 2 \quad \sec x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

42. $\sec x \csc x = 2 \csc x$

$$\sec x \csc x - 2 \csc x = 0$$

$$\csc x(\sec x - 2) = 0$$

$$\csc x = 0 \quad \text{or} \quad \sec x - 2 = 0$$

$$\text{No solution} \quad \sec x = 2$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

44. $\sec x + \tan x = 1$

$$(\sec x + \tan x)(\sec x - \tan x) = \sec x - \tan x$$

$$\sec^2 x - \tan^2 x = \sec x - \tan x$$

$$1 = \sec x - \tan x$$

$$\text{Hence, } \sec x + \tan x = \sec x - \tan x \Rightarrow \tan x = 0.$$

$$\sec x = 1, \tan x = 0 \Rightarrow x = 0$$

43. $2 \sin x + \csc x = 0$

$$2 \sin x + \frac{1}{\sin x} = 0$$

$$2 \sin^2 x + 1 = 0$$

$$\text{Since } 2 \sin^2 x + 1 > 0, \text{ there are no solutions.}$$

45. $\cos x + \sin x \tan x = 2$

$$\cos x + \frac{\sin^2 x}{\cos x} = 2$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$$

$$\frac{1}{\cos x} = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

46. $\sin^2 x + \cos x + 1 = 0$

$$(1 - \cos^2 x) + \cos x + 1 = 0$$

$$\cos^2 x - \cos x - 2 = 0$$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x = 2, \text{ Impossible}$$

$$\cos x + 1 = 0 \Rightarrow x = \pi$$

47. $\sec^2 x + \tan x = 3$

$$(1 + \tan^2 x) + \tan x = 3$$

$$\tan^2 x + \tan x - 2 = 0$$

$$(\tan x + 2)(\tan x - 1) = 0$$

$$\tan x = -2 \quad \text{or} \quad \tan x = 1$$

$$x \approx 2.0344, 5.1760 \quad x = \frac{\pi}{4}, \frac{5\pi}{4}$$

48. $2 \cos^2 x + \cos x - 1 = (2 \cos x - 1)(\cos x + 1) = 0$

$$2 \cos x = 1 \quad \text{or} \quad \cos x = -1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

49. $2 \sin^2 x + 3 \sin x + 1 = 0$

$$y = 2 \sin^2 x + 3 \sin x + 1$$

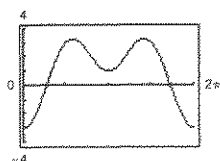
$$x \approx 3.6652, 5.7596, 4.7124$$

50. $2 \sec^2 x + \tan^2 x - 3 = 0$

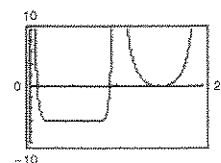
$$y = \frac{2}{\cos^2 x} + \tan^2 x - 3$$

$$x \approx 0.5236, 2.6180, 3.6652, 5.7596$$

51. $y = 4 \sin^2 x - 2 \cos x - 1$
 $x \approx 0.8614, 5.4218$

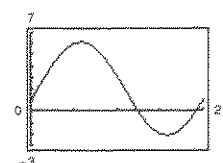


52. $y = \frac{1}{\sin^2 x} - \frac{3}{\sin x} - 4$
 $x \approx 0.2527, 2.8889, 4.7124$



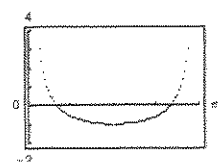
53. $y = \csc x + \cot x - 1 = \frac{1}{\sin x} + \frac{\cos x}{\sin x} - 1$
 $x \approx 1.5708, \left(\frac{\pi}{2}\right)$

54. $y = 4 \sin x - \cos x + 2$
 $x \approx 3.8930, 6.0217$

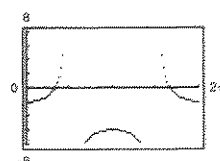


55. $\frac{\cos x \cot x}{1 - \sin x} = 3$

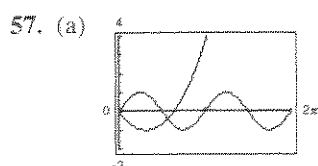
Graph $y = \frac{\cos x}{(1 - \sin x) \tan x} - 3$.

 The solutions are approximately
 $x \approx 0.5236, x \approx 2.6180$.


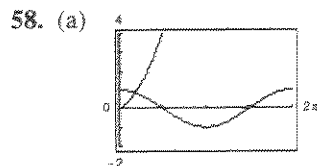
56. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} - 4 = 0$



$x \approx 1.0472, 5.2360$

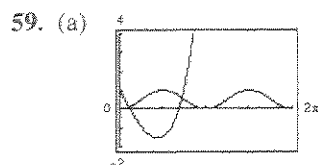


(b) $\sin 2x = x^2 - 2x$

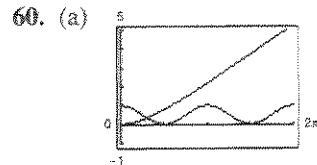


(b) $\cos x = x + x^2$

 (c) Points of intersection: $(0, 0), (1.7757, -0.3984)$

 (c) Points of intersection:
 $(-1.2512, 0.3142)$ (outside interval),
 $(0.5500, 0.8525)$


(b) $\sin^2 x = e^x - 4x$



(b) $\cos^2 x = e^{-x} + x - 1$

 (c) Points of intersection:
 $(0.3194, 0.0986), (2.2680, 0.5878)$

 (c) Points of intersection:
 $(0.9510, 0.3374)$, and
 $(-0.8266, -0.4589)$ (outside interval)

61. $\cos \frac{x}{4} = 0$

$$\frac{x}{4} = \frac{\pi}{2} + 2n\pi \quad \text{or} \quad \frac{x}{4} = \frac{3\pi}{2} + 2n\pi$$

$$x = 2\pi + 8n\pi \quad \text{or} \quad x = 6\pi + 8n\pi$$

Combining, $x = 2\pi + 4n\pi$.

62. $\sin \frac{x}{2} = 0$

$$\frac{x}{2} = n\pi$$

$$x = 2n\pi$$

63. $\sin 4x = 1$

$$4x = \frac{\pi}{2} + 2n\pi$$

$$x = \frac{\pi}{8} + \frac{n\pi}{2}$$

64. $\cos 2x = -1$

$$2x = \pi + 2n\pi$$

$$x = \frac{\pi}{2} + n\pi$$

65. $\sin 2x = -\frac{\sqrt{3}}{2}$

$$2x = \frac{4\pi}{3} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{2\pi}{3} + n\pi \quad x = \frac{5\pi}{6} + n\pi$$

66. $\sec 4x = 2$

$$4x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad 4x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2} \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

67. $2 \sin^2 2x = 1$

$$\sin^2 2x = \frac{1}{2}$$

$$\sin 2x = \pm \frac{\sqrt{2}}{2}$$

$$2x = \frac{\pi}{4} + \frac{n\pi}{2}$$

$$x = \frac{\pi}{8} + \frac{n\pi}{4}$$

68. $\tan^2 3x = 3$

$$\tan 3x = \pm \sqrt{3}$$

$$3x = \frac{\pi}{3} + n\pi \quad \text{or} \quad 3x = \frac{2\pi}{3} + n\pi$$

$$x = \frac{\pi}{9} + \frac{n\pi}{3} \quad \text{or} \quad x = \frac{2\pi}{9} + \frac{n\pi}{3}$$

69. $\tan 3x(\tan x - 1) = 0$

$$\tan 3x = 0 \quad \text{or} \quad \tan x - 1 = 0$$

$$3x = n\pi \quad \text{or} \quad x = \frac{\pi}{4} + n\pi$$

$$x = \frac{n\pi}{3} \quad x = \frac{\pi}{4} + n\pi$$

70. $\cos 2x(2 \cos x + 1) = 0$

$$\cos 2x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$2x = \frac{\pi}{2} + n\pi \quad \cos x = -\frac{1}{2}$$

$$x = \frac{\pi}{4} + \frac{n\pi}{2} \quad x = \frac{2\pi}{3} + 2n\pi$$

$$x = \frac{4\pi}{3} + 2n\pi$$

71. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

$$\frac{x}{2} = \frac{\pi}{4} + 2n\pi \quad \text{or} \quad \frac{x}{2} = \frac{7\pi}{4} + 2n\pi$$

$$x = \frac{\pi}{2} + 4n\pi \quad x = \frac{7\pi}{2} + 4n\pi$$

72. $\tan \frac{x}{3} = 1$

$$\frac{x}{3} = \frac{\pi}{4} + n\pi$$

$$x = \frac{3\pi}{4} + 3n\pi$$

73. $y = \sin \frac{\pi x}{2} + 1$

From the graph in the textbook we see that the curve has x -intercepts at $x = -1$ and at $x = 3$.

74. $y = \sin \pi x + \cos \pi x$

From the graph in the textbook, we see that the curve has x -intercepts at $x = -0.25, 0.75, 1.75$, and 2.75 .

75. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$

From the graph in the textbook, we see that the curve has x -intercepts at $x = \pm 2$.

76. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$

From the graph in the textbook, we see that the curve has x -intercepts at $x = -2, 2$.

77. $2 \cos x - \sin x = 0$

Graph $y_1 = 2 \cos x - \sin x$ and estimate the zeros.

$$x \approx 1.1071, 4.2487$$

78. $y = 2 \sin x + \cos x$

$$x \approx 2.6779, 5.8195$$

79. $x \tan x - 1 = 0$

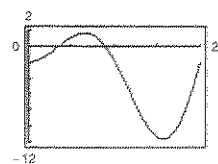
Graph $y_1 = x \tan x - 1$ and estimate the zeros.

$$x \approx 0.8603, 3.4256$$

80. $2x \sin x - 2 = 0$

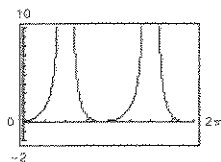
$$y = 2x \sin x - 2$$

$$x \approx 1.1142, 2.7726$$



81. $\sec^2 x + 0.5 \tan x - 1 = 0$

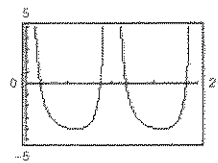
$$\text{Graph } y_1 = \frac{1}{(\cos x)^2} + 0.5 \tan x - 1.$$



$$x = 0, x \approx 2.6779, 3.1416, 5.8195$$

82. $\csc^2 x + 0.5 \cot x - 5 = 0$

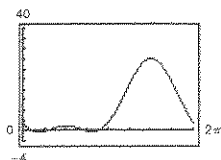
$$y_1 = \left(\frac{1}{\sin x}\right)^2 + \frac{1}{2 \tan x} - 5$$



$$x \approx 0.5153, 2.7259, 3.6569, 5.8675$$

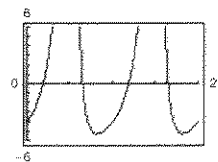
83. $12 \sin^2 x - 13 \sin x + 3 = 0$

$$\text{Graph } y_1 = 12 \sin^2 x - 13 \sin x + 3.$$



$$x \approx 0.3398, 0.8481, 2.2935, 2.8018$$

84. $3 \tan^2 x + 4 \tan x - 4 = 0$



$$x \approx 0.5880, 2.0344, 3.7296, 5.1760$$

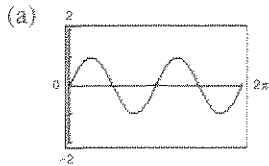
$$85. 3 \tan^2 x + 5 \tan x - 4 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$x \approx -1.154, 0.534$$

$$87. 4 \cos^2 x - 2 \sin x + 1 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$x \approx 1.110$$

$$89. f(x) = \sin 2x$$



Maxima: (0.7854, 1), (3.9270, 1)

Minima: (2.3562, -1), (5.4978, -1)

$$(b) 2 \cos 2x = 0$$

$$\cos 2x = 0$$

$$2x = \frac{\pi}{2} + n\pi$$

$$x = \frac{\pi}{4} + \frac{n\pi}{2}$$

The zeros are 0.7854, 2.3562, 3.9270, and 5.4978.

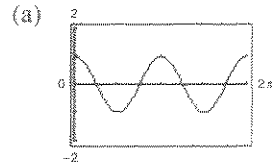
$$86. y = \cos^2 x - 2 \cos x - 1 = 0, [0, \pi]$$

$$x \approx 1.998$$

$$88. y = 2 \sec^2 x + \tan x - 6 = 0, \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$x \approx -1.035, 0.870$$

$$90. f(x) = \cos 2x$$



Maxima: (0, 1), (3.1416, 1), (6.2832, 1)

Minima: (1.5708, -1), (4.7124, -1)

$$(b) -2 \sin 2x = 0$$

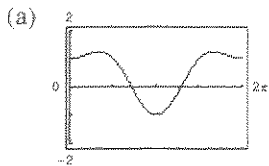
$$\sin 2x = 0$$

$$2x = n\pi$$

$$x = \frac{n\pi}{2}$$

The zeros are 0, 1.5708, 3.1416, 4.7124, and 6.2832.

$$91. f(x) = \sin^2 x + \cos x$$



Maxima: (1.0472, 1.25), (5.2360, 1.25)

Minima: (3.1416, -1)

$$(b) 2 \sin x \cos x - \sin x = 0$$

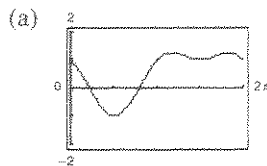
$$\sin x(2 \cos x - 1) = 0$$

$$\sin x = 0 \Rightarrow x = n\pi$$

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi$$

The zeros are 1.0472, 3.1416, and 5.2360.

$$92. f(x) = \cos^2 x - \sin x$$



Maxima: (3.6652, 1.25), (5.7596, 1.25)

Minima: (1.5708, -1)

$$(b) -2 \sin x \cos x - \cos x = 0$$

$$\cos x(2 \sin x + 1) = 0$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + n\pi$$

$$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} + 2n\pi, \frac{11\pi}{6} + 2n\pi$$

The zeros are 1.5708, 3.6652, 5.7596, and 4.7124.

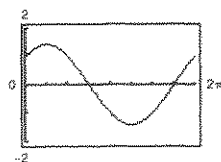
93. (a) $f(x) = \sin x + \cos x$

Maximum:

$(0.7854, 1.4142)$

Minimum:

$(3.9270, -1.4142)$



(b) $\cos x - \sin x = 0$

$\cos x = \sin x$

$1 = \frac{\sin x}{\cos x}$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

$f\left(\frac{5\pi}{4}\right) = \sin \frac{5\pi}{4} + \cos \frac{5\pi}{4}$

$= -\sin \frac{\pi}{4} + \left(-\cos \frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

Therefore, the maximum point in the interval $[0, 2\pi)$ is $(\pi/4, \sqrt{2})$ and the minimum point is $(5\pi/4, -\sqrt{2})$.

95. $f(x) = \tan \frac{\pi x}{4}$

$\tan 0 = 0$, but 0 is not positive. By graphing

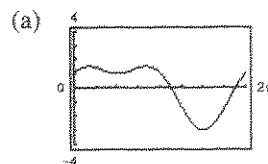
$y = \tan \frac{\pi x}{4} - x,$

you see that the smallest positive fixed point is $x = 1$.

97. $f(x) = \cos \frac{1}{x}$

(a) The domain of $f(x)$ is all real numbers except 0.(c) As $x \rightarrow 0$, $f(x)$ oscillates between -1 and 1 .(e) The greatest solution appears to occur at $x \approx 0.6366$.

94. $y = 2 \sin x + \cos 2x$

Maximum: $(0.5236, 1.5), (2.6180, 1.5)$ Minimum: $(4.7124, -3.0)$

(b) $2 \cos x - 4 \sin x \cos x = 0$

$2 \cos x(1 - 2 \sin x) = 0$

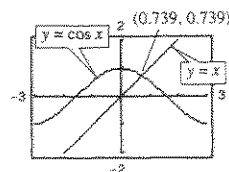
$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 - 2 \sin x = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

The zeros are 0.5236, 2.618, 4.712 and 1.571.

The first three correspond to the values in (a).

96. Graph $y = \cos x$ and $y = x$ on the same set of axes. Their point of intersection gives the value of c such that $f(c) = c \Rightarrow \cos c = c$.



$c \approx 0.739$

(b) The graph has y-axis symmetry and a horizontal asymptote at $y = 1$.(d) There are an infinite number of solutions in the interval $[-1, 1]$.

$\frac{1}{x} = \frac{\pi}{2} + n\pi = \frac{\pi + 2n\pi}{2} \Rightarrow x = \frac{2}{\pi(2n+1)}$

98. $f(x) = \frac{\sin x}{x}$

(a) Domain: all real numbers except $x = 0$.

(b) The graph has y -axis symmetry.

Horizontal asymptote: $y = 0$

(c) As $x \rightarrow 0$, $f(x) \rightarrow 1$.

(d) $\sin x/x = 0$ has four solutions in the interval $[-8, 8]$.

$$(\sin x)\left(\frac{1}{x}\right) = 0$$

$$\sin x = 0$$

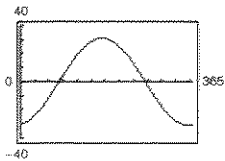
$$x = -2\pi, -\pi, \pi, 2\pi$$

99. $S = 74.50 - 43.75 \cos \frac{\pi t}{6}$

t	1	2	3	4	5	6	7	8	9	10	11	12
S	36.6	52.6	74.5	96.4	112.4	118.3	112.4	96.4	74.5	52.6	36.6	30.8

$S > 100$ for $t = 5, 6, 7$ (May, June, July)

100. $D = 31 \sin\left(\frac{2\pi}{365}t - 1.4\right)$



$D > 20^\circ$ for $123 \leq t \leq 223$ days

101.

$$y = \frac{1}{12}(\cos 8t - 3 \sin 8t)$$

$$\frac{1}{12}(\cos 8t - 3 \sin 8t) = 0$$

$$\cos 8t = 3 \sin 8t$$

$$\frac{1}{3} = \tan 8t$$

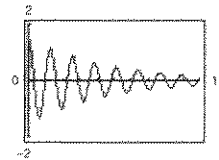
$$8t = 0.32175 + n\pi$$

$$t = 0.04 + \frac{n\pi}{8}$$

In the interval $0 \leq t \leq 1$, $t = 0.04, 0.43$, and 0.83 second.

102. $y_1 = 1.56e^{-0.22t} \cos 4.9t$ intersects $y_2 = -1$ at $t \approx 1.96$ (and other points).

The displacement does not exceed one foot from equilibrium after $t = 1.96$ seconds.



103. $r = \frac{1}{32}v_0^2 \sin 2\theta$

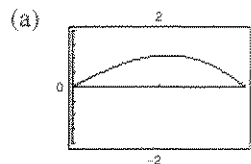
$$300 = \frac{1}{32}(100)^2 \sin 2\theta$$

$$\sin 2\theta = 0.96$$

$$2\theta \approx 1.287 \quad \text{or} \quad 2\theta \approx \pi - 1.287 \approx 1.855$$

$$\theta \approx 0.6435 \approx 37^\circ \quad \text{or} \quad \theta \approx 0.9275 \approx 53^\circ$$

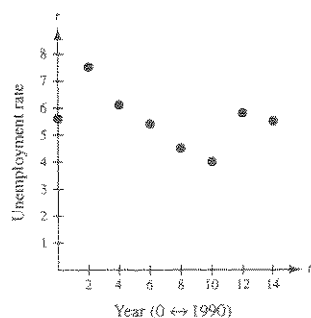
104. $A = 2x \cos x$, $0 \leq x \leq \frac{\pi}{2}$



The maximum area of $A \approx 1.12$ occurs when $x \approx 0.86$.

(b) $A \geq 1$ for $0.6 < x < 1.1$

105. (a)



(b) Models 1 and 2 are both good fits, but model 1 seems better.

(c) The constant term 5.45 gives the average unemployment rate, 5.45%.

(d) The length is approximately one period

$$\frac{2\pi}{0.47} \approx 13.37 \text{ years.}$$

$$(e) r = 1.24 \sin(0.47t + 0.40) + 5.45 = 5.0$$

Using a graphing utility, $t \approx 19.99 \approx 20$, or 2010.

$$106. f(x) = 3 \sin(0.6x - 2)$$

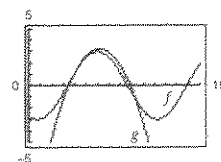
$$(a) \text{ Zero: } \sin(0.6x - 2) = 0$$

$$0.6x - 2 = 0$$

$$0.6x = 2$$

$$x = \frac{2}{0.6} = \frac{10}{3}$$

$$(b) g(x) = -0.45x^2 + 5.52x - 13.70$$

For $3.5 \leq x \leq 6$ the approximation appears to be good. Answers will vary.

$$(c) -0.45x^2 + 5.52x - 13.70 = 0$$

$$x = \frac{-5.52 \pm \sqrt{(5.52)^2 - 4(-0.45)(-13.70)}}{2(-0.45)}$$

$$x \approx 3.46, 8.81$$

The zero of g on $[0, 6]$ is 3.46. The zero is close to the zero $\frac{10}{3} \approx 3.33$ of f .107. False. $\sin x - x = 0$ has one solution, $x = 0$.108. False. There might not be periodicity, as in the equation $\sin(x^2) = 0$.109. False. The equation has no solution because $-1 \leq \sin x \leq 1$.

110. Answers will vary.

$$111. 124^\circ = 124^\circ \left(\frac{\pi}{180^\circ} \right) \approx 2.164 \text{ radians}$$

$$112. 486^\circ = 486^\circ \left(\frac{\pi}{180^\circ} \right) \approx 8.482 \text{ radians}$$

$$113. -0.41^\circ = -0.41^\circ \left(\frac{\pi}{180^\circ} \right) \approx -0.007 \text{ radian}$$

$$114. -210.55^\circ = -210.55^\circ \left(\frac{\pi}{180^\circ} \right) \approx -3.675 \text{ radians}$$

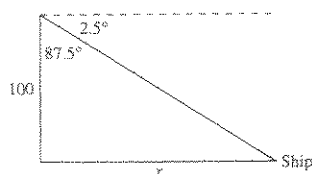
$$115. \tan 30^\circ = \frac{14}{x} \Rightarrow x = \frac{14}{\tan 30^\circ} = \frac{14}{\sqrt{3}/3} \approx 24.249$$

$$116. \sin 70^\circ = \frac{x}{10} \Rightarrow x = 10 \cdot \sin 70^\circ \approx 9.397 \approx 9.4$$

$$117. \tan 87.5^\circ = \frac{x}{100}$$

$$x = 100 \tan 87.5^\circ$$

$$\approx 2290.4 \text{ feet} \approx 0.43 \text{ mile}$$



118. Answers will vary.

Section 5.4 Sum and Difference Formulas

- You should memorize the sum and difference formulas.

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- You should be able to use these formulas to find the values of the trigonometric functions of angles whose sums or differences are special angles.
- You should be able to use these formulas to solve trigonometric equations.

Vocabulary Check

1. $\sin u \cos v - \cos u \sin v$

2. $\cos u \cos v - \sin u \sin v$

3. $\frac{\tan u + \tan v}{1 - \tan u \tan v}$

4. $\sin u \cos v + \cos u \sin v$

5. $\cos u \cos v + \sin u \sin v$

6. $\frac{\tan u - \tan v}{1 + \tan u \tan v}$

1. (a) $\cos(240^\circ - 0^\circ) = \cos(240^\circ) = -\frac{1}{2}$

(b) $\cos(240^\circ) - \cos 0^\circ = -\frac{1}{2} - 1 = -\frac{3}{2}$

2. (a) $\sin(405^\circ + 120^\circ) = \sin 405^\circ \cos 120^\circ + \cos 405^\circ \sin 120^\circ$

$$= \frac{\sqrt{2}}{2} \left(-\frac{1}{2} \right) + \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

(b) $\sin 405^\circ + \sin 120^\circ = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{3}}{2}$

3. (a) $\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3}$

$$= \frac{\sqrt{2}}{2} \left(\frac{1}{2} \right) - \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} \right)$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

4. (a) $\sin\left(\frac{2\pi}{3} + \frac{5\pi}{6}\right) = \sin\left(\frac{9\pi}{6}\right) = -1$

(b) $\sin \frac{2\pi}{3} + \sin \frac{5\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$

(b) $\cos \frac{\pi}{4} + \cos \frac{\pi}{3} = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2} + 1}{2}$

5. (a) $\sin(315^\circ - 60^\circ) = \sin 315^\circ \cos 60^\circ - \cos 315^\circ \sin 60^\circ = -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{-\sqrt{2} - \sqrt{6}}{4}$

(b) $\sin 315^\circ - \sin 60^\circ = -\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} = \frac{-\sqrt{2} - \sqrt{3}}{2}$

6. (a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\left(\frac{5\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{1}{2}$

(b) $\sin \frac{7\pi}{6} - \sin \frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{-1 - \sqrt{3}}{2}$

$$7. \sin 105^\circ = \sin(60^\circ + 45^\circ)$$

$$= \sin 60^\circ \cos 45^\circ + \sin 45^\circ \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{2}}{4}(\sqrt{3} + 1)$$

$$\cos 105^\circ = \cos(60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4}(1 - \sqrt{3})$$

$$\tan 105^\circ = \tan(60^\circ + 45^\circ)$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{-2} = -2 - \sqrt{3}$$

$$9. \sin 195^\circ = \sin(225^\circ - 30^\circ)$$

$$= \sin 225^\circ \cos 30^\circ - \sin 30^\circ \cos 225^\circ$$

$$= -\sin 45^\circ \cos 30^\circ + \sin 30^\circ \cos 45^\circ$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(1 - \sqrt{3})$$

$$\cos 195^\circ = \cos(225^\circ - 30^\circ)$$

$$= \cos 225^\circ \cos 30^\circ + \sin 225^\circ \sin 30^\circ$$

$$= -\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$$

$$\tan 195^\circ = \tan(225^\circ - 30^\circ)$$

$$= \frac{\tan 225^\circ - \tan 30^\circ}{1 + \tan 225^\circ \tan 30^\circ}$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 - (\sqrt{3}/3)}{1 + (\sqrt{3}/3)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

$$8. 165^\circ = 135^\circ + 30^\circ$$

$$\sin 165^\circ = \sin(135^\circ + 30^\circ)$$

$$= \sin 135^\circ \cos 30^\circ + \sin 30^\circ \cos 135^\circ$$

$$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)$$

$$\cos 165^\circ = \cos(135^\circ + 30^\circ)$$

$$= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$$

$$= -\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$$

$$\tan 165^\circ = \tan(135^\circ + 30^\circ)$$

$$= \frac{\tan 135^\circ + \tan 30^\circ}{1 - \tan 135^\circ \tan 30^\circ}$$

$$= \frac{-\tan 45^\circ + \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ}$$

$$= \frac{-1 + (\sqrt{3}/3)}{1 + (\sqrt{3}/3)} = -2 + \sqrt{3}$$

$$10. 255^\circ = 300^\circ - 45^\circ$$

$$\sin 255^\circ = \sin(300^\circ - 45^\circ)$$

$$= \sin 300^\circ \cos 45^\circ - \cos 300^\circ \sin 45^\circ$$

$$= \left(-\frac{\sqrt{3}}{2}\right) \frac{\sqrt{2}}{2} - \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 255^\circ = \cos(300^\circ - 45^\circ)$$

$$= \cos 300^\circ \cos 45^\circ + \sin 300^\circ \sin 45^\circ$$

$$= \frac{1}{2} \left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) \frac{\sqrt{2}}{2} = \frac{-\sqrt{6} + \sqrt{2}}{4}$$

$$\tan 255^\circ = \tan(300^\circ - 45^\circ)$$

$$= \frac{\tan 300^\circ - \tan 45^\circ}{1 + \tan 300^\circ \tan 45^\circ}$$

$$= \frac{-\sqrt{3} - 1}{1 + (-\sqrt{3})(1)}$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$$

$$\begin{aligned}
 11. \quad \sin \frac{11\pi}{12} &= \sin\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) \\
 &= \sin \frac{3\pi}{4} \cos \frac{\pi}{6} + \sin \frac{\pi}{6} \cos \frac{3\pi}{4} \\
 &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4}(\sqrt{3} - 1) \\
 \cos \frac{11\pi}{12} &= \cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) \\
 &= \cos \frac{3\pi}{4} \cos \frac{\pi}{6} - \sin \frac{3\pi}{4} \sin \frac{\pi}{6} \\
 &= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\
 &= -\frac{\sqrt{2}}{4}(\sqrt{3} + 1) \\
 \tan \frac{11\pi}{12} &= \tan\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) \\
 &= \frac{\tan(3\pi/4) + \tan(\pi/6)}{1 - \tan(3\pi/4) \tan(\pi/6)} \\
 &= \frac{-1 + (\sqrt{3}/3)}{1 - (-1)(\sqrt{3}/3)} \\
 &= \frac{-3 + \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\
 &= \frac{-12 + 6\sqrt{3}}{6} = -2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{17\pi}{12} &= \frac{7\pi}{6} + \frac{\pi}{4} \\
 \sin \frac{17\pi}{12} &= \sin\left(\frac{7\pi}{6} + \frac{\pi}{4}\right) \\
 &= \sin \frac{7\pi}{6} \cos \frac{\pi}{4} + \cos \frac{7\pi}{6} \sin \frac{\pi}{4} \\
 &= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} \\
 &= \frac{-\sqrt{6} - \sqrt{2}}{4} \\
 \cos \frac{17\pi}{12} &= \cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right) \\
 &= \cos \frac{7\pi}{6} \cos \frac{\pi}{4} - \sin \frac{7\pi}{6} \sin \frac{\pi}{4} \\
 &= \left(-\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} \\
 &= \frac{\sqrt{2} - \sqrt{6}}{4} \\
 \tan \frac{17\pi}{12} &= \tan\left(\frac{7\pi}{6} + \frac{\pi}{4}\right) \\
 &= \frac{\tan \frac{7\pi}{6} + \tan \frac{\pi}{4}}{1 - \tan \frac{7\pi}{6} \tan \frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}(1)} \\
 &= \frac{\sqrt{3} + 3}{3 - \sqrt{3}} = 2 + \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad -\frac{\pi}{12} &= \frac{\pi}{6} - \frac{\pi}{4} \\
 \sin\left(-\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{6} - \frac{\pi}{4}\right) \\
 &= \sin \frac{\pi}{6} \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \cos \frac{\pi}{6} \\
 &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}(1 - \sqrt{3}) \\
 \cos\left(-\frac{\pi}{12}\right) &= \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) \\
 &= \cos \frac{\pi}{6} \cos \frac{\pi}{4} + \sin \frac{\pi}{6} \sin \frac{\pi}{4} \\
 &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} + 1)
 \end{aligned}$$

$$\begin{aligned}
 \tan\left(-\frac{\pi}{12}\right) &= \tan\left(\frac{\pi}{6} - \frac{\pi}{4}\right) \\
 &= \frac{\tan(\pi/6) - \tan(\pi/4)}{1 + \tan(\pi/6) \tan(\pi/4)} \\
 &= \frac{(\sqrt{3}/3) - 1}{1 + (\sqrt{3}/3)} = \frac{\sqrt{3} - 3}{\sqrt{3} + 3} \cdot \frac{\sqrt{3} - 3}{\sqrt{3} - 3} \\
 &= \frac{12 - 6\sqrt{3}}{-6} = -2 + \sqrt{3}
 \end{aligned}$$

$$14. -\frac{19\pi}{12} = \frac{2\pi}{3} - \frac{9\pi}{4}$$

$$\begin{aligned}\sin\left(-\frac{19\pi}{12}\right) &= \sin\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right) \\ &= \sin\frac{2\pi}{3}\cos\frac{9\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{9\pi}{4} \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right) \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos\left(-\frac{19\pi}{12}\right) &= \cos\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right) \\ &= \cos\frac{2\pi}{3}\cos\frac{9\pi}{4} + \sin\frac{2\pi}{3}\sin\frac{9\pi}{4} \\ &= \left(-\frac{1}{2}\right) \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\tan\left(-\frac{19\pi}{12}\right) &= \tan\left(\frac{2\pi}{3} - \frac{9\pi}{4}\right) = \frac{\tan\frac{2\pi}{3} - \tan\frac{9\pi}{4}}{1 + \tan\frac{2\pi}{3}\tan\frac{9\pi}{4}} \\ &= \frac{-\sqrt{3} - 1}{1 + (-\sqrt{3})(1)} \\ &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= 2 + \sqrt{3}\end{aligned}$$

$$16. 15^\circ = 45^\circ - 30^\circ$$

$$\begin{aligned}\sin 15^\circ &= \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}(\sqrt{3} - 1)}{4} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)\end{aligned}$$

$$\begin{aligned}\cos 15^\circ &= \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}(\sqrt{3} + 1)}{4} = \frac{\sqrt{2}}{4}(\sqrt{3} + 1)\end{aligned}$$

$$\begin{aligned}\tan 15^\circ &= \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \\ &= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\ &= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}\end{aligned}$$

$$15. \sin 75^\circ = \sin(30^\circ + 45^\circ)$$

$$\begin{aligned}&= \sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4}(1 + \sqrt{3})\end{aligned}$$

$$\begin{aligned}\cos 75^\circ &= \cos(30^\circ + 45^\circ) \\ &= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\sqrt{3} - 1)\end{aligned}$$

$$\begin{aligned}\tan 75^\circ &= \tan(30^\circ + 45^\circ) \\ &= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} \\ &= \frac{(\sqrt{3}/3) + 1}{1 - (\sqrt{3}/3)} = \frac{\sqrt{3} + 3}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}} \\ &= \frac{6\sqrt{3} + 12}{6} = \sqrt{3} + 2\end{aligned}$$

17. -225° is coterminal with 135° , and lies in Quadrant II.

$$\sin(-225^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(-225^\circ) = -\frac{\sqrt{2}}{2}$$

$$\tan(-225^\circ) = -1$$

18. $-165^\circ = -135^\circ - 30^\circ$

$$\begin{aligned} \text{(a) } \sin(-165^\circ) &= \sin(-135^\circ - 30^\circ) \\ &= \sin(-135^\circ) \cos 30^\circ - \cos(-135^\circ) \sin 30^\circ \\ &= \left(-\frac{\sqrt{2}}{2}\right) \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2}\right) \frac{1}{2} = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} \text{(b) } \cos(-165^\circ) &= \cos(-135^\circ - 30^\circ) \\ &= \cos(-135^\circ) \cos 30^\circ + \sin(-135^\circ) \sin 30^\circ \\ &= \left(-\frac{\sqrt{2}}{2}\right) \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right) \frac{1}{2} = \frac{-\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{(c) } \tan(-165^\circ) &= \tan(-135^\circ - 30^\circ) \\ &= \frac{\tan(-135^\circ) - \tan 30^\circ}{1 + \tan(-135^\circ) \tan 30^\circ} \\ &= \frac{1 - \sqrt{3}/3}{1 + 1(\sqrt{3}/3)} \\ &= \frac{3 - \sqrt{3}}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} \\ &= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3} \end{aligned}$$

19. $\frac{13\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$

$$\begin{aligned} \sin \frac{13\pi}{12} &= \sin\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{3\pi}{4} \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cos \frac{3\pi}{4} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} \cos \frac{13\pi}{12} &= \cos\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \cos \frac{3\pi}{4} \cos \frac{\pi}{3} - \sin \frac{3\pi}{4} \sin \frac{\pi}{3} \\ &= \left(-\frac{\sqrt{2}}{2}\right) \left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \tan \frac{13\pi}{12} &= \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right) = \frac{\tan(3\pi/4) + \tan(\pi/3)}{1 - \tan(3\pi/4) \tan(\pi/3)} \\ &= \frac{(-1) + \sqrt{3}}{1 - (-1)\sqrt{3}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} \end{aligned}$$

$$20. \frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$$

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{\pi}{4} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{5\pi}{12}\right) &= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6} \\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\tan\left(\frac{5\pi}{12}\right) = \frac{\sin(5\pi/12)}{\cos(5\pi/12)} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = 2 + \sqrt{3}$$

$$21. -\frac{7\pi}{12} = \frac{\pi}{6} - \frac{3\pi}{4}$$

$$\begin{aligned}\sin\left(-\frac{7\pi}{12}\right) &= \sin\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4}\cos\frac{\pi}{6} \\ &= \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

$$\begin{aligned}\cos\left(-\frac{7\pi}{12}\right) &= \cos\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{3\pi}{4} + \sin\frac{\pi}{6}\sin\frac{3\pi}{4} \\ &= \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}\end{aligned}$$

$$\begin{aligned}\tan\left(-\frac{7\pi}{12}\right) &= \tan\left(\frac{\pi}{6} - \frac{3\pi}{4}\right) = \frac{\tan(\pi/6) - \tan(3\pi/4)}{1 + \tan(\pi/6)\tan(3\pi/4)} \\ &= \frac{(\sqrt{3}/3) - (-1)}{1 + (\sqrt{3}/3)(-1)} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = 2 + \sqrt{3}\end{aligned}$$

$$22. -\frac{13\pi}{12} = \frac{-3\pi}{4} - \frac{\pi}{3}$$

$$\begin{aligned}\sin\left(\frac{-13\pi}{12}\right) &= \sin\left(\frac{-3\pi}{4} - \frac{\pi}{3}\right) = \sin\left(\frac{-3\pi}{4}\right)\cos\frac{\pi}{3} - \cos\left(\frac{-3\pi}{4}\right)\sin\frac{\pi}{3} \\ &= \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right) - \left(\frac{-\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{-13\pi}{12}\right) &= \cos\left(\frac{-3\pi}{4} - \frac{\pi}{3}\right) = \cos\left(\frac{-3\pi}{4}\right)\cos\frac{\pi}{3} + \sin\left(\frac{-3\pi}{4}\right)\sin\frac{\pi}{3} \\ &= \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\frac{\sqrt{3}}{2} = -\frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

$$\tan\left(\frac{-13\pi}{12}\right) = \frac{\sin\left(\frac{-13\pi}{12}\right)}{\cos\left(\frac{-13\pi}{12}\right)} = \frac{\sqrt{6} - \sqrt{2}}{-\sqrt{6} - \sqrt{2}} = \sqrt{3} - 2$$

$$23. \cos 60^\circ \cos 20^\circ - \sin 60^\circ \sin 20^\circ = \cos(60^\circ + 20^\circ) = \cos 80^\circ$$

$$24. \sin 110^\circ \cos 80^\circ + \cos 110^\circ \sin 80^\circ = \sin(110^\circ + 80^\circ) = \sin(190^\circ)$$

$$25. \frac{\tan 325^\circ - \tan 86^\circ}{1 + \tan 325^\circ \tan 86^\circ} = \tan(325^\circ - 86^\circ) = \tan 239^\circ \quad 26. \frac{\tan 154^\circ - \tan 49^\circ}{1 + \tan 154^\circ \tan 49^\circ} = \tan(154^\circ - 49^\circ) = \tan 105^\circ$$

$$27. \sin 3.5 \cos 1.2 - \cos 3.5 \sin 1.2 = \sin(3.5 - 1.2) = \sin 2.3$$

$$28. \cos 0.96 \cos 0.42 + \sin 0.96 \sin 0.42 = \cos(0.96 - 0.42) = \cos(0.54)$$

$$29. \cos \frac{\pi}{9} \cos \frac{\pi}{7} - \sin \frac{\pi}{9} \sin \frac{\pi}{7} = \cos\left(\frac{\pi}{9} + \frac{\pi}{7}\right) = \cos\left(\frac{16\pi}{63}\right)$$

$$30. \sin \frac{4\pi}{9} \cos \frac{\pi}{8} + \cos \frac{4\pi}{9} \sin \frac{\pi}{8} = \sin\left(\frac{4\pi}{9} + \frac{\pi}{8}\right) = \sin\left(\frac{41\pi}{72}\right)$$

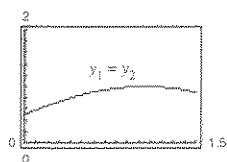
$$31. y_1 = \sin\left(\frac{\pi}{6} + x\right)$$

$$= \sin \frac{\pi}{6} \cos x + \sin x \cdot \cos \frac{\pi}{6}$$

$$= \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$$

$$= \frac{1}{2}(\cos x + \sqrt{3} \sin x) = y_2$$

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y ₁	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384
y ₂	0.6621	0.7978	0.9017	0.9696	0.9989	0.9883	0.9384

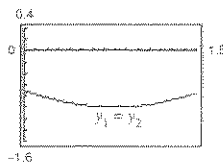


x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y ₁	-0.8335	-0.9266	-0.9829	-0.9999	-0.9771	-0.9153	-0.8170
y ₂	-0.8335	-0.9266	-0.9829	-0.9999	-0.9771	-0.9153	-0.8170

$$y_1 = \cos\left(\frac{5\pi}{4} - x\right) = \cos \frac{5\pi}{4} \cos x + \sin \frac{5\pi}{4} \sin x$$

$$= -\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x$$

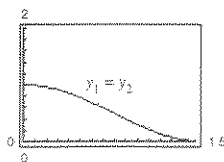
$$= -\frac{\sqrt{2}}{2}(\cos x + \sin x) = y_2$$



$$33. y_1 = \cos(x + \pi) \cos(x - \pi)$$

$$= (\cos x \cdot \cos \pi - \sin x \cdot \sin \pi)[\cos x \cos \pi + \sin x \sin \pi]$$

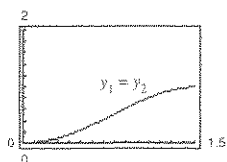
$$= [-\cos x][-\cos x] = \cos^2 x = y_2$$



x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y ₁	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289
y ₂	0.9605	0.8484	0.6812	0.4854	0.2919	0.1313	0.0289

$$34. y_1 = \sin(x + \pi) \sin(x - \pi)$$

$$= [\sin x \cos \pi + \sin \pi \cos x][\sin x \cos \pi - \sin \pi \cos x] = [-\sin x][-\sin x] = \sin^2 x = y_2$$



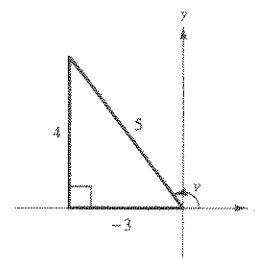
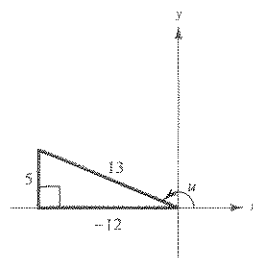
x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1	0.0395	0.1516	0.3188	0.5146	0.7081	0.8687	0.9711
y_2	0.0395	0.1516	0.3188	0.5146	0.7081	0.8687	0.9711

For Exercises 35–38,

$$\sin u = \frac{5}{13} \text{ and } u \text{ in Quadrant II} \Rightarrow \cos u = -\frac{12}{13}$$

$$\cos v = -\frac{3}{5} \text{ and } v \text{ in Quadrant II} \Rightarrow \sin v = \frac{4}{5}$$

$$\tan u = -\frac{5}{12} \text{ and } \tan v = -\frac{4}{3}$$



$$35. \sin(u + v) = \sin u \cos v + \sin v \cos u$$

$$= \frac{5}{13} \left(-\frac{3}{5} \right) + \frac{4}{5} \left(-\frac{12}{13} \right) = -\frac{63}{65}$$

$$36. \cos(v - u) = \cos v \cos u + \sin v \sin u$$

$$= \left(-\frac{3}{5} \right) \left(-\frac{12}{13} \right) + \left(\frac{4}{5} \right) \left(\frac{5}{13} \right) = \frac{56}{65}$$

$$37. \tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{\left(-\frac{5}{12} \right) - \left(\frac{4}{3} \right)}{1 - \left(-\frac{5}{12} \right) \left(-\frac{4}{3} \right)}$$

$$= \frac{-63/36}{16/36} = -\frac{63}{16}$$

$$38. \sin(u - v) = \sin u \cos v - \sin v \cos u$$

$$= \frac{5}{13} \left(-\frac{3}{5} \right) - \left(\frac{4}{5} \right) \left(-\frac{12}{13} \right)$$

$$= \frac{33}{65}$$

For Exercises 39–42, $\sin u = -\frac{8}{17}$, $\cos u = -\frac{15}{17}$, $\sin v = -\frac{3}{5}$, $\cos v = -\frac{4}{5}$, $\tan u = \frac{8}{15}$, $\tan v = \frac{3}{4}$.

$$39. \cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$= \left(-\frac{15}{17} \right) \left(-\frac{4}{5} \right) - \left(-\frac{8}{17} \right) \left(-\frac{3}{5} \right) = \frac{36}{85}$$

$$40. \tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{(8/15) + (3/4)}{1 - (8/15)(3/4)} = \frac{32 + 45}{60 - 24} = \frac{77}{36}$$

$$41. \sin(v - u) = \sin v \cos u - \cos v \sin u$$

$$= \left(-\frac{3}{5} \right) \left(-\frac{15}{17} \right) - \left(-\frac{4}{5} \right) \left(-\frac{8}{17} \right) = \frac{13}{85}$$

$$42. \cos(u - v) = \cos u \cos v + \sin u \sin v$$

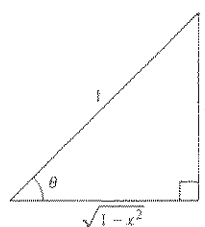
$$= \left(-\frac{15}{17} \right) \left(-\frac{4}{5} \right) + \left(-\frac{8}{17} \right) \left(-\frac{3}{5} \right) = \frac{84}{85}$$

$$43. \sin(\arcsin x + \arccos x) = \sin(\arcsin x) \cos(\arccos x) + \sin(\arccos x) \cos(\arcsin x)$$

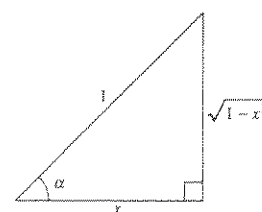
$$= x \cdot x + \sqrt{1 - x^2} \cdot \sqrt{1 - x^2}$$

$$= x^2 + 1 - x^2$$

$$= 1$$



$\theta = \arcsin x$



$\alpha = \arccos x$

44. Let: $u = \arccos x$ and $v = \arcsin x$

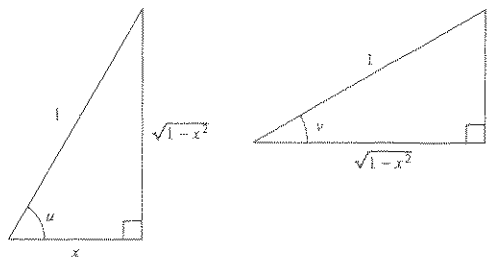
$$\cos u = x$$

$$\sin v = x$$

$$\cos(\arccos x - \arcsin x) = \cos(\arccos x) \cos(\arcsin x) + \sin(\arccos x) \sin(\arcsin x)$$

$$= x\sqrt{1-x^2} + \sqrt{1-x^2}x$$

$$= 2x\sqrt{1-x^2}$$



45. Let: $u = \arctan 2x$ and $v = \arccos x$

$$\tan u = 2x$$

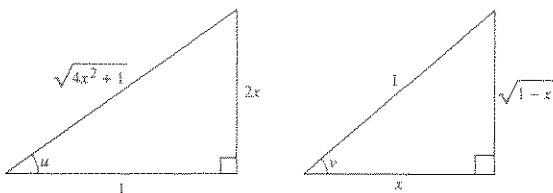
$$\cos v = x$$

$$\sin(\arctan 2x - \arccos x) = \sin(u - v)$$

$$= \sin u \cos v - \cos u \sin v$$

$$= \frac{2x}{\sqrt{4x^2+1}}(x) - \frac{1}{\sqrt{4x^2+1}}(\sqrt{1-x^2})$$

$$= \frac{2x^2 - \sqrt{1-x^2}}{\sqrt{4x^2+1}}$$



46. Let: $u = \arcsin x$ and $v = \arctan 2x$

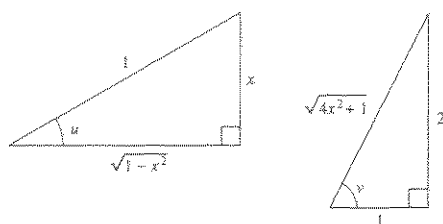
$$\sin u = x$$

$$\tan v = 2x$$

$$\cos(\arcsin x - \arctan 2x) = \cos(\arcsin x) \cos(\arctan 2x) + \sin(\arcsin x) \sin(\arctan 2x)$$

$$= \sqrt{1-x^2} \frac{1}{\sqrt{4x^2+1}} + x \frac{2x}{\sqrt{4x^2+1}}$$

$$= \frac{2x^2 + \sqrt{1-x^2}}{\sqrt{4x^2+1}}$$



47. $\sin^{-1} 1 = \frac{\pi}{2}$ because $\sin \frac{\pi}{2} = 1$.

$\cos^{-1} 1 = 0$ because $\cos 0 = 1$.

$$\sin(\sin^{-1} 1 + \cos^{-1} 1) = \sin\left(\frac{\pi}{2} + 0\right) = 1$$

48. $\sin^{-1}(-1) = -\frac{\pi}{2}$ and $\cos^{-1} 0 = \frac{\pi}{2}$

$$\begin{aligned} \cos(\sin^{-1}(-1) + \cos^{-1} 0) &= \cos\left(-\frac{\pi}{2} + \frac{\pi}{2}\right) \\ &= \cos 0 = 1 \end{aligned}$$

49. $\sin^{-1} 1 = \frac{\pi}{2}$ and $\cos^{-1}(-1) = \pi$

$$\sin(\sin^{-1} 1 - \cos^{-1}(-1)) = \sin\left(\frac{\pi}{2} - \pi\right)$$

$$= \sin\left(-\frac{\pi}{2}\right) = -1$$

50. $\cos^{-1}(-1) = \pi$ and $\cos^{-1} 1 = 0$

$$\cos(\cos^{-1}(-1) - \cos^{-1} 1) = \cos(\pi - 0) = -1$$

$$51. \sin^{-1} \frac{1}{2} = \frac{\pi}{6} \text{ and } \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$$

$$\begin{aligned} \sin\left(\sin^{-1} \frac{1}{2} - \cos^{-1} \frac{1}{2}\right) &= \sin\left(\frac{\pi}{6} - \frac{\pi}{3}\right) \\ &= \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2} \end{aligned}$$

$$53. \sin^{-1} 0 = 0 \text{ and } \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$\tan\left(\sin^{-1} 0 + \sin^{-1} \frac{1}{2}\right) = \tan\left(0 + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

$$55. \sin^{-1}(-1) = -\frac{\pi}{2}$$

$$\sin\left(\frac{\pi}{2} + \sin^{-1}(-1)\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = \sin 0 = 0$$

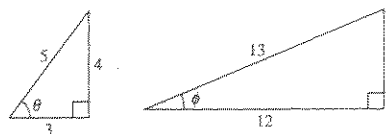
$$57. \sin^{-1} 1 = \frac{\pi}{2}$$

$$\cos\left(\pi + \sin^{-1} 1\right) = \cos\left(\pi + \frac{\pi}{2}\right) = \cos \frac{3\pi}{2} = 0$$

$$59. \text{ Let } \theta = \cos^{-1} \frac{3}{5} \Rightarrow \cos \theta = \frac{3}{5}.$$

$$\text{ Let } \phi = \sin^{-1} \frac{5}{13} \Rightarrow \sin \phi = \frac{5}{13}.$$

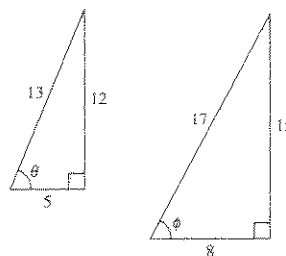
$$\begin{aligned} \sin\left(\cos^{-1} \frac{3}{5} - \sin^{-1} \frac{5}{13}\right) &= \sin(\theta - \phi) \\ &= \sin \theta \cos \phi - \cos \theta \sin \phi \\ &= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) = \frac{33}{65} \end{aligned}$$



$$60. \text{ Let } \theta = \sin^{-1} \frac{12}{13} \Rightarrow \sin \theta = \frac{12}{13}.$$

$$\text{ Let } \phi = \cos^{-1} \frac{8}{17} \Rightarrow \cos \phi = \frac{8}{17}.$$

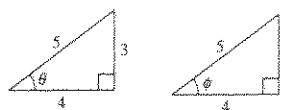
$$\begin{aligned} \cos\left(\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{8}{17}\right) &= \cos(\theta + \phi) \\ &= \cos \theta \cos \phi - \sin \theta \sin \phi \\ &= \left(\frac{5}{13}\right)\left(\frac{8}{17}\right) - \left(\frac{12}{13}\right)\left(\frac{15}{17}\right) = -\frac{140}{221} \end{aligned}$$



$$61. \text{ Let } \theta = \tan^{-1} \frac{3}{4} \Rightarrow \tan \theta = \frac{3}{4}.$$

$$\text{ Let } \phi = \sin^{-1} \frac{3}{5} \Rightarrow \sin \phi = \frac{3}{5}.$$

$$\begin{aligned} \sin\left(\tan^{-1} \frac{3}{4} + \sin^{-1} \frac{3}{5}\right) &= \sin(\theta + \phi) \\ &= \sin \theta \cos \phi + \sin \phi \cos \theta \\ &= \frac{3}{5}\left(\frac{4}{5}\right) + \frac{3}{5}\left(\frac{4}{5}\right) = \frac{24}{25} \end{aligned}$$



Note: $\theta = \phi$

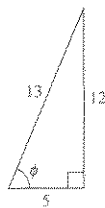
$$62. \text{ Let } \theta = \sin^{-1} \frac{4}{5} \Rightarrow \sin \theta = \frac{4}{5}.$$

$$\text{Let } \phi = \cos^{-1} \frac{5}{13} \Rightarrow \cos \phi = \frac{5}{13}.$$

$$\tan\left(\sin^{-1} \frac{4}{5} - \cos^{-1} \frac{5}{13}\right) = \tan(\theta - \phi)$$

$$= \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$$

$$= \frac{(4/3) - (12/5)}{1 + (4/3) \cdot (12/5)} = \frac{-16}{63}$$



$$63. \sin\left(\frac{\pi}{2} + x\right) = \sin \frac{\pi}{2} \cos x + \sin x \cos \frac{\pi}{2}$$

$$= (1) \cos x + 0 = \cos x$$

$$64. \sin(3\pi - x) = \sin 3\pi \cos x - \cos 3\pi \sin x$$

$$= (0) \cos x - (-1) \sin x = \sin x$$

$$65. \tan(x + \pi) - \tan(\pi - x) = \frac{\tan x + \tan \pi}{1 - \tan x \cdot \tan \pi} - \frac{\tan \pi - \tan x}{1 + \tan \pi \tan x}$$

$$= \frac{\tan x}{1} - \left(-\frac{\tan x}{1}\right) = 2 \tan x$$

$$66. \tan\left(\frac{\pi}{4} - \theta\right) = \frac{\tan(\pi/4) - \tan \theta}{1 + \tan(\pi/4) \tan \theta} = \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$67. \sin(x + y) + \sin(x - y) = \sin x \cos y + \sin y \cos x + \sin x \cos y - \sin y \cos x = 2 \sin x \cos y$$

$$68. \cos(x + y) + \cos(x - y) = \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y = 2 \cos x \cos y$$

$$69. \cos(x + y) \cos(x - y) = [\cos x \cos y - \sin x \sin y][\cos x \cos y + \sin x \sin y]$$

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y = \cos^2 x (1 - \sin^2 y) - \sin^2 x \sin^2 y$$

$$= \cos^2 x - \sin^2 y (\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 y$$

$$70. \sin(x + y) \sin(x - y) = [\sin x \cos y + \cos x \sin y][\sin x \cos y - \cos x \sin y]$$

$$= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y = \sin^2 x (1 - \sin^2 y) - \cos^2 x \sin^2 y$$

$$= \sin^2 x - \sin^2 y (\sin^2 x + \cos^2 x) = \sin^2 x - \sin^2 y$$

$$71. \sin\left(x + \frac{\pi}{3}\right) + \sin\left(x - \frac{\pi}{3}\right) = 1$$

$$\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3} + \sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3} = 1$$

$$2 \sin x (0.5) = 1$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$72. \quad \cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$$

$$\left(\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}\right) - \left(\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6}\right) = 1$$

$$-2 \sin x \sin \frac{\pi}{6} = 1$$

$$-2 \sin x \left(\frac{1}{2}\right) = 1$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

$$73. \quad \tan(x + \pi) + 2 \sin(x + \pi) = 0$$

$$\frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2(\sin x \cos \pi + \cos x \sin \pi) = 0$$

$$\frac{\tan x + 0}{1 - \tan x(0)} + 2[\sin x(-1) + \cos x(0)] = 0$$

$$\frac{\tan x}{1} - 2 \sin x = 0$$

$$\frac{\sin x}{\cos x} = 2 \sin x$$

$$\sin x = 2 \sin x \cos x$$

$$\sin x(1 - 2 \cos x) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = 0, \pi \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$74. \quad 2 \sin\left(x + \frac{\pi}{2}\right) + 3 \tan(\pi - x) = 0$$

$$2\left[\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2}\right] + 3 \tan(-x) = 0$$

$$2 \cos x - 3 \frac{\sin x}{\cos x} = 0$$

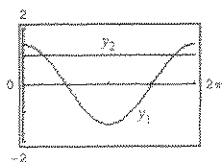
$$2 \cos^2 x - 3 \sin x = 0$$

$$2(1 - \sin^2 x) - 3 \sin x = 0$$

$$2 \sin^2 x + 3 \sin x - 2 = 0$$

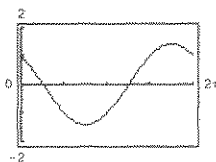
$$(2 \sin x - 1)(\sin x + 2) \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

75. Graph $y_1 = \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right)$ and $y_2 = 1$.



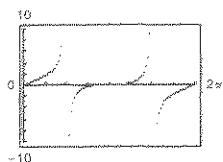
$$x \approx 0.7854, 5.4978$$

76. $\sin\left(x + \frac{\pi}{2}\right) - \cos\left(x + \frac{3\pi}{2}\right) = 0$



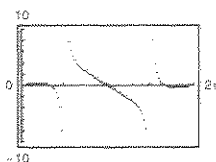
$$x \approx 0.7854, 3.9270$$

77. $\tan(x + \pi) - \cos\left(x + \frac{\pi}{2}\right) = 0$



Answers: 0.0, 3.1416 ($x = 0, \pi$)

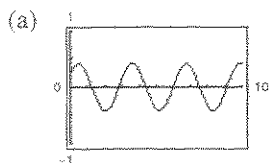
78. $\tan(\pi - x) + 2 \cos\left(x + \frac{3\pi}{2}\right) = 0$



$$x = 0, 1.0472, 3.1416, 5.2360$$

$$\begin{aligned} 79. \quad y_1 + y_2 &= A \cos 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + A \cos 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \\ &= A \left[\cos\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi x}{\lambda}\right) + \sin\left(\frac{2\pi t}{T}\right) \sin\left(\frac{2\pi x}{\lambda}\right) \right] + A \left[\cos\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi x}{\lambda}\right) - \sin\left(\frac{2\pi t}{T}\right) \sin\left(\frac{2\pi x}{\lambda}\right) \right] \\ &= 2A \cos\left(\frac{2\pi t}{T}\right) \cos\left(\frac{2\pi x}{\lambda}\right) \end{aligned}$$

80. $y = \frac{1}{3} \sin 2t + \frac{1}{4} \cos 2t$



(c) Amplitude: $\frac{5}{12}$

(b) $a = \frac{1}{3}, b = \frac{1}{4}, B = 2$

$$C = \arctan \frac{b}{a} = \arctan \frac{3}{4} \approx 0.6435$$

$$y \approx \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2} \sin(2t + 0.6435) = \frac{5}{12} \sin(2t + 0.6435)$$

(d) Frequency: $\frac{1}{\text{period}} = \frac{B}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$

81. False. See page 380.

82. True. $\sin\left(x - \frac{11\pi}{2}\right) = \sin x \cos \frac{11\pi}{2} - \cos x \sin \frac{11\pi}{2} = 0 - \cos x(-1) = \cos x$

83. $\cos(n\pi + \theta) = \cos n\pi \cos \theta - \sin n\pi \sin \theta$

$$= (-1)^n(\cos \theta) - (0)(\sin \theta)$$

$$= (-1)^n(\cos \theta), \text{ where } n \text{ is an integer.}$$

84. $\sin(n\pi + \theta) = \sin n\pi \cos \theta + \sin \theta \cos n\pi$

$$= (0)(\cos \theta) + (\sin \theta)(-1)^n$$

$$= (-1)^n(\sin \theta), \text{ where } n \text{ is an integer.}$$

$$85. C = \arctan \frac{b}{a} \Rightarrow \tan C = \frac{b}{a} \Rightarrow \sin C = \frac{b}{\sqrt{a^2 + b^2}}, \cos C = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\sqrt{a^2 + b^2} \sin(B\theta + C) = \sqrt{a^2 + b^2} \left(\sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} \cdot \cos B\theta \right) = a \sin B\theta + b \cos B\theta$$

$$86. C = \arctan \frac{a}{b} \Rightarrow \sin C = \frac{a}{\sqrt{a^2 + b^2}}, \cos C = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\begin{aligned} \sqrt{a^2 + b^2} \cos(B\theta - C) &= \sqrt{a^2 + b^2} \left(\cos B\theta \cdot \frac{b}{\sqrt{a^2 + b^2}} + \sin B\theta \cdot \frac{a}{\sqrt{a^2 + b^2}} \right) \\ &= b \cos B\theta + a \sin B\theta \\ &= a \sin B\theta + b \cos B\theta \end{aligned}$$

$$87. \sin \theta + \cos \theta$$

$$a = 1, b = 1, B = 1$$

$$(a) C = \arctan \frac{b}{a} = \arctan 1 = \frac{\pi}{4}$$

$$\begin{aligned} \sin \theta + \cos \theta &= \sqrt{a^2 + b^2} \sin(B\theta + C) \\ &= \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) \end{aligned}$$

$$(b) C = \arctan \frac{a}{b} = \arctan 1 = \frac{\pi}{4}$$

$$\begin{aligned} \sin \theta + \cos \theta &= \sqrt{a^2 + b^2} \cos(B\theta - C) \\ &= \sqrt{2} \cos\left(\theta - \frac{\pi}{4}\right) \end{aligned}$$

$$88. 3 \sin 2\theta + 4 \cos 2\theta; a = 3, b = 4, B = 2$$

$$(a) C = \arctan \frac{b}{a} = \arctan \frac{4}{3} \approx 0.9273$$

$$\begin{aligned} 3 \sin 2\theta + 4 \cos 2\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C) \\ &\approx 5 \sin(2\theta + 0.9273) \end{aligned}$$

$$(b) C = \arctan \frac{a}{b} = \arctan \frac{3}{4} \approx 0.6435$$

$$\begin{aligned} 3 \sin 2\theta + 4 \cos 2\theta &= \sqrt{a^2 + b^2} \cos(B\theta - C) \\ &\approx 5 \cos(2\theta - 0.6435) \end{aligned}$$

$$89. 12 \sin 3\theta + 5 \cos 3\theta; a = 12, b = 5, B = 3$$

$$(a) C = \arctan \frac{b}{a} = \arctan \frac{5}{12} \approx 0.3948$$

$$\begin{aligned} 12 \sin 3\theta + 5 \cos 3\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C) \\ &\approx 13 \sin(3\theta + 0.3948) \end{aligned}$$

$$(b) C = \arctan \frac{a}{b} = \arctan \frac{12}{5} \approx 1.1760$$

$$\begin{aligned} 12 \sin 3\theta + 5 \cos 3\theta &= \sqrt{a^2 + b^2} \cos(B\theta - C) \\ &\approx 13 \cos(3\theta - 1.1760) \end{aligned}$$

$$90. \sin 2\theta - \cos 2\theta; a = 1, b = -1, B = 2$$

$$(a) C = \arctan \frac{b}{a} = \arctan(-1) = -\frac{\pi}{4}$$

$$\begin{aligned} \sin 2\theta - \cos 2\theta &= \sqrt{a^2 + b^2} \sin(B\theta + C) \\ &= \sqrt{2} \sin\left(2\theta - \frac{\pi}{4}\right) \end{aligned}$$

(b) Because $b > 0$ in the formula, we write the given expression as

$$-(-\sin 2\theta + \cos 2\theta); a = -1, b = 1, B = 2,$$

$$C = \arctan\left(\frac{a}{b}\right) = \arctan(-1) = -\frac{\pi}{4}.$$

Hence,

$$\begin{aligned} -(-\sin 2\theta + \cos 2\theta) &= -\sqrt{a^2 + b^2} \cos(B\theta - C) \\ &= -\sqrt{2} \cos\left(2\theta + \frac{\pi}{4}\right). \end{aligned}$$

$$91. C = \arctan \frac{b}{a} = \frac{\pi}{2} \Rightarrow a = 0$$

$$\sqrt{a^2 + b^2} = 2 \Rightarrow b = 2$$

$$B = 1$$

$$2 \sin\left(\theta + \frac{\pi}{2}\right) = (0)(\sin \theta) + (2)(\cos \theta) = 2 \cos \theta$$

$$92. C = -\frac{\pi}{4} = \arctan\left(\frac{a}{b}\right) \Rightarrow \frac{a}{b} = -1$$

$$\Rightarrow a = -1, b = 1$$

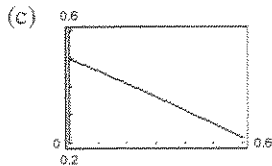
$$\sqrt{a^2 + b^2} = \sqrt{2}$$

Hence, $B = 1$ and

$$\begin{aligned} 5 \cos\left(\theta + \frac{\pi}{4}\right) &= \frac{5}{\sqrt{2}} \sqrt{2} \cos\left[\theta - \left(-\frac{\pi}{4}\right)\right] \\ &= \frac{5}{\sqrt{2}} [-\sin \theta + \cos \theta] \\ &= -\frac{5}{\sqrt{2}} \sin \theta + \frac{5}{\sqrt{2}} \cos \theta \\ &= -\frac{5\sqrt{2}}{2} \sin \theta + \frac{5\sqrt{2}}{2} \cos \theta. \end{aligned}$$

$$\begin{aligned} 93. \frac{\sin(x+h) - \sin x}{h} &= \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} \\ &= \frac{\cos x \sin h}{h} - \frac{\sin x(1 - \cos h)}{h} \end{aligned}$$

94. (a) Domains of f and g are all real numbers, $h \neq 0$.



(b)

h	0.01	0.02	0.05	0.1	0.2	0.5
$f(h)$	0.4957	0.4913	0.4781	0.4559	0.4104	0.2674
$g(h)$	0.4957	0.4913	0.4781	0.4559	0.4104	0.2674

(d) As $h \rightarrow 0$, $f \rightarrow \frac{1}{2}$ and $g \rightarrow \frac{1}{2}$. In fact, $f = g$.

95. From the figure, it appears that $u + v = w$. Assume that u , v , and w are all in Quadrant I. From the figure:

$$\tan u = \frac{s}{3s} = \frac{1}{3}$$

$$\tan v = \frac{s}{2s} = \frac{1}{2}$$

$$\tan w = \frac{s}{s} = 1$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} = \frac{(1/3) + (1/2)}{1 - (1/3)(1/2)} = \frac{5/6}{1 - (1/6)} = 1 = \tan w.$$

Thus, $\tan(u+v) = \tan w$. Because u , v , and w are all in Quadrant I, we have

$$\arctan[\tan(u+v)] = \arctan[\tan w]$$

$$u + v = w.$$

$$96. (a) \sin(u + v + w) = \sin u \cos(v + w) + \cos u \sin(v + w)$$

$$= \sin u [\cos v \cos w - \sin v \sin w] + \cos u [\sin v \cos w + \sin w \cos v]$$

$$= \sin u \cos v \cos w - \sin u \sin v \sin w + \cos u \sin v \cos w + \cos u \sin w \cos v$$

$$(b) \tan(u + v + w) = \frac{\tan(u + v) + \tan w}{1 - \tan(u + v) \tan w} = \frac{\left[\frac{\tan u + \tan v}{1 - \tan u \tan v} \right] + \tan w}{1 - \left[\frac{\tan u + \tan v}{1 - \tan u \tan v} \right] \tan w}$$

$$= \frac{\tan u + \tan v + \tan w(1 - \tan u \tan v)}{(1 - \tan u \tan v) - (\tan u + \tan v) \tan w} = \frac{\tan u + \tan v + \tan w - \tan u \tan v \tan w}{1 - \tan u \tan v - \tan u \tan w - \tan v \tan w}$$

$$97. x = 0: y = -\frac{1}{2}(0 - 10) + 14 = 5 + 14 = 19$$

$$y\text{-intercept: } (0, 19)$$

$$y = 0: 0 = -\frac{1}{2}(x - 10) + 14$$

$$= -\frac{1}{2}x + 19 \Rightarrow x = 38$$

$$x\text{-intercept: } (38, 0)$$

$$98. y = 0: x^2 - 3x - 40 = (x - 8)(x + 5) = 0$$

$$x\text{-intercepts: } (8, 0), (-5, 0)$$

$$x = 0 \Rightarrow y = -40$$

$$y\text{-intercept: } (0, -40)$$

$$99. x = 0: |2(0) - 9| - 5 = 9 - 5 = 4$$

$$y\text{-intercept: } (0, 4)$$

$$y = 0: |2x - 9| = 5 \Rightarrow x = 7, 2$$

$$x\text{-intercepts: } (2, 0), (7, 0)$$

$$100. y = 0: 2x\sqrt{x+7} = 0 \Rightarrow x = 0, -7$$

$$x\text{-intercepts: } (0, 0), (-7, 0)$$

$$x = 0 \Rightarrow y = 0$$

$$y\text{-intercept: } (0, 0)$$

$$101. \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \text{ because } \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}.$$

$$102. \arctan(-\sqrt{3}) = -\frac{\pi}{3} \text{ because } \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}.$$

$$103. \arcsin 1 = \frac{\pi}{2} \text{ because } \sin \frac{\pi}{2} = 1.$$

$$104. \arctan 0 = 0$$

Section 5.5 Multiple-Angle and Product-to-Sum Formulas

■ You should know the following double-angle formulas.

$$(a) \sin 2u = 2 \sin u \cos u$$

$$(b) \cos 2u = \cos^2 u - \sin^2 u$$

$$(c) \tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$= 2 \cos^2 u - 1$$

$$= 1 - 2 \sin^2 u$$

■ You should be able to reduce the power of a trigonometric function.

$$(a) \sin^2 u = \frac{1 - \cos 2u}{2}$$

$$(b) \cos^2 u = \frac{1 + \cos 2u}{2}$$

$$(c) \tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

■ You should be able to use the half-angle formulas.

$$(a) \sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$(b) \cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$(c) \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

—CONTINUED—

—CONTINUED—

■ You should be able to use the product-sum formulas.

$$(a) \sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$(b) \cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$(c) \sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$(d) \cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

■ You should be able to use the sum-product formulas.

$$(a) \sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$(b) \sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$(c) \cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$(d) \cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

Vocabulary Check

1. $2 \sin u \cos u$

2. $\frac{1 + \cos 2u}{2}$

3. $\cos 2u$

4. $\tan \frac{u}{2}$

5. $\frac{2 \tan u}{1 - \tan^2 u}$

6. $\frac{1}{2}[\cos(u - v) + \cos(u + v)]$

7. $\sin^2 u$

8. $\cos \frac{u}{2}$

9. $\frac{1}{2}[\sin(u + v) + \sin(u - v)]$

10. $2 \sin\left(\frac{u + v}{2}\right) \cos\left(\frac{u - v}{2}\right)$

1. (a) $\sin \theta = \frac{3}{5}$

(b) $\cos \theta = \frac{4}{5}$

$$(c) \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$(d) \sin 2\theta = 2 \sin \theta \cos \theta$$

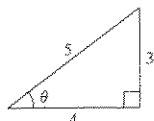
$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

$$(e) \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{24}{7}$$

$$(f) \sec 2\theta = \frac{1}{\cos 2\theta} = \frac{25}{7}$$

$$(g) \csc 2\theta = \frac{1}{\sin 2\theta} = \frac{25}{24}$$

$$(h) \cot 2\theta = \frac{1}{\tan 2\theta} = \frac{7}{24}$$



2. (a) $\sin \theta = \frac{12}{13}$

(b) $\cos \theta = \frac{5}{13}$

$$(c) \sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$$

$$(d) \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

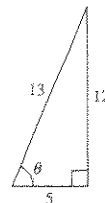
$$= \frac{25}{169} - \frac{144}{169} = \frac{-119}{169}$$

$$(e) \tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120}{119}$$

$$(f) \cot 2\theta = \frac{1}{\tan 2\theta} = \frac{-119}{120}$$

$$(g) \sec 2\theta = \frac{1}{\cos 2\theta} = \frac{-169}{119}$$

$$(h) \csc 2\theta = \frac{1}{\sin 2\theta} = \frac{169}{120}$$



3. $\sin 2x - \sin x = 0$

Solutions: 0, 1.047, 3.142, 5.236

Analytically:

$$\sin 2x - \sin x = 0$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x(2 \cos x - 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad 2 \cos x - 1 = 0$$

$$x = 0, \pi$$

$$\cos x = \frac{1}{2}$$

$$x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

5. $4 \sin x \cos x = 1$

 $x \approx 0.2618, 1.3090, 3.4034, 4.4506$

Analytically:

$$4 \sin x \cos x = 1$$

$$2 \sin(2x) = 1$$

$$\sin(2x) = \frac{1}{2}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

4. $\sin 2x + \cos x = 0$

Solutions: 1.5708, 3.6652, 4.7124, 5.7596

Analytically:

$$\sin 2x + \cos x = 0$$

$$2 \sin x \cos x + \cos x = 0$$

$$\cos x(2 \sin x + 1) = 0$$

$$\cos x = 0 \quad \sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

6. $\sin 2x \sin x = \cos x$

 $x \approx 0.7854, 1.5708, 2.3562, 3.9270, 4.7124, 5.4978$

Analytically:

$$2 \sin x \cos x \sin x - \cos x = 0$$

$$\cos x(2 \sin^2 x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad 2 \sin^2 x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

7. $\cos 2x - \cos x = 0$

 $x \approx 0, 2.094, 4.189, (6.283 \text{ not in interval})$

Analytically:

$$\cos 2x - \cos x = 0$$

$$2 \cos^2 x - 1 - \cos x = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2}, \quad \cos x = 1$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0, (2\pi \text{ not in interval})$$

8. $\tan 2x - \cot x = 0$

$$x \approx 0.5236, 1.5708, 2.6180, 3.6652, 4.7124, 5.7596$$

Analytically:

$$\frac{2 \tan x}{1 - \tan^2 x} = \cot x$$

$$2 \tan x = \cot x(1 - \tan^2 x)$$

$$2 \tan x = \cot x - \cot x \tan^2 x$$

$$2 \tan x = \cot x - \tan x$$

$$3 \tan x = \cot x$$

$$3 \tan x - \cot x = 0$$

$$3 \tan x - \frac{1}{\tan x} = 0$$

$$\frac{3 \tan^2 x - 1}{\tan x} = 0$$

$$\frac{1}{\tan x}(3 \tan^2 x - 1) = 0$$

$$\cot x(3 \tan^2 x - 1) = 0$$

$$\cot x = 0 \quad \text{or} \quad 3 \tan^2 x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

9. Solutions: 0, 1.571, 3.142, 4.712

$$\sin 4x = -2 \sin 2x$$

$$\sin 4x + 2 \sin 2x = 0$$

$$2 \sin 2x \cos 2x + 2 \sin 2x = 0$$

$$2 \sin 2x(\cos 2x + 1) = 0$$

$$2 \sin 2x = 0 \quad \text{or} \quad \cos 2x + 1 = 0$$

$$\sin 2x = 0 \quad \cos 2x = -1$$

$$2x = n\pi \quad 2x = \pi + 2n\pi$$

$$x = \frac{n}{2}\pi \quad x = \frac{\pi}{2} + n\pi$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

10. $(\sin 2x + \cos 2x)^2 = 1$

$$x \approx 0.0, 0.7854, 1.5708, 2.3562, 3.1416, 3.9270,$$

$$4.7124, 5.4978$$

Analytically:

$$\sin^2 2x + 2 \sin 2x \cos 2x + \cos^2 2x = 1$$

$$2 \sin 2x \cos 2x = 0$$

$$\sin 4x = 0$$

$$4x = n\pi$$

$$x = \frac{n\pi}{4}$$

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4},$$

$$\pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

11. $\cos 2x + \sin x = 0$

Using a graphing utility, $x \approx 1.5708, 3.6652, 5.7596$.

Algebraically:

$$\cos 2x + \sin x = 0$$

$$1 - 2 \sin^2 x + \sin x = 0$$

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin x = 1 \Rightarrow x = \frac{\pi}{2}$$

12. $\tan 2x - 2 \cos x = 0$

Using a graphing utility,

$$x \approx 0.5236, 1.5708, 2.6180, 4.7124.$$

Algebraically:

$$\tan 2x = 2 \cos x$$

$$\frac{2 \tan x}{1 - \tan^2 x} = 2 \cos x$$

$$\frac{\sin x}{\cos x} = \cos x \left(1 - \frac{\sin^2 x}{\cos^2 x} \right)$$

$$\left(\text{Note: } \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \right)$$

$$\sin x = \cos^2 x - \sin^2 x$$

$$\sin x = 1 - 2 \sin^2 x$$

$$2 \sin^2 x + \sin x - 1 = 0$$

$$(2 \sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$$

14. $\cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi$, Quadrant II

$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{\sqrt{45}}{7} \right) \left(-\frac{2}{7} \right) = -\frac{12\sqrt{5}}{49}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{49} - \frac{45}{49} = -\frac{41}{49}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(-\sqrt{45}/2)}{1 - (45/4)} = \frac{-\sqrt{45}}{(4 - 45)/4} = \frac{12\sqrt{5}}{41}$$

15. $\tan u = \frac{1}{2}, \pi < u < \frac{3\pi}{2} \Rightarrow \sin u = -\frac{1}{\sqrt{5}}$ and $\cos u = -\frac{2}{\sqrt{5}}$

$$\sin 2u = 2 \sin u \cos u = 2 \left(-\frac{1}{\sqrt{5}} \right) \left(-\frac{2}{\sqrt{5}} \right) = \frac{4}{5}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(-\frac{2}{\sqrt{5}} \right)^2 - \left(-\frac{1}{\sqrt{5}} \right)^2 = \frac{3}{5}$$

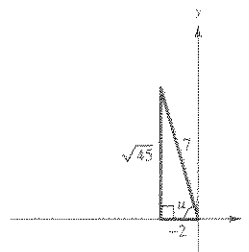
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(1/2)}{1 - (1/4)} = \frac{4}{3}$$

13. $\sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2} \Rightarrow \cos u = \frac{4}{5}$

$$\sin 2u = 2 \sin u \cos u = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(3/4)}{1 - (9/16)} = \frac{24}{7}$$

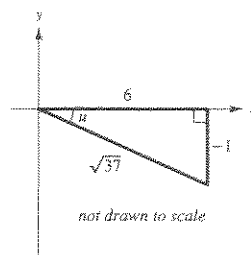


16. $\cot u = -6, \frac{3\pi}{2} < u < 2\pi$, Quadrant IV

$$\sin 2u = 2 \sin u \cos u = 2 \left(-\frac{1}{\sqrt{37}} \right) \left(\frac{6}{\sqrt{37}} \right) = -\frac{12}{37}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{36}{37} - \frac{1}{37} = \frac{35}{37}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{2(-1/6)}{1 - (-1/6)^2} = \frac{-2/6}{35/36} = -\frac{12}{35}$$



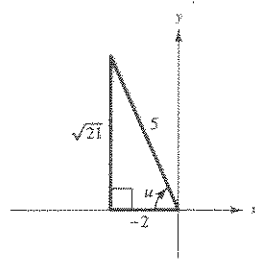
17. $\sec u = -\frac{5}{2}, \frac{\pi}{2} < u < \pi$

$$\cos u = -\frac{2}{5} \Rightarrow \sin u = \frac{\sqrt{21}}{5}$$

$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{\sqrt{21}}{5} \right) \left(-\frac{2}{5} \right) = -\frac{4\sqrt{21}}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{25} - \frac{21}{25} = -\frac{17}{25}$$

$$\begin{aligned} \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \\ &= \frac{2(\sqrt{21}/-2)}{1 - (21/4)} = \frac{-\sqrt{21}}{-17/4} = \frac{4\sqrt{21}}{17} \end{aligned}$$



18. $\csc u = 3, \frac{\pi}{2} < u < \pi$

$$\sin u = \frac{1}{3}, \cos u = -\frac{2\sqrt{2}}{3}, \tan u = \frac{-1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{1}{3} \right) \left(-\frac{2\sqrt{2}}{3} \right) = -\frac{4\sqrt{2}}{9}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u} = \frac{(-2\sqrt{2})/4}{1 - (1/8)} = \frac{-4\sqrt{2}}{7}$$

19. $8 \sin x \cos x = 4(2 \sin x \cos x) = 4 \sin 2x$

20. $4 \sin x \cos x + 1 = 2 \sin 2x + 1$

21. $6 - 12 \sin^2 x = 6(1 - 2 \sin^2 x) = 6 \cos 2x$

22. $(\cos x + \sin x)(\cos x - \sin x) = \cos^2 x - \sin^2 x$
 $= \cos 2x$

23. $\begin{aligned} \cos^4 x &= (\cos^2 x)(\cos^2 x) = \left(\frac{1 + \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right) = \frac{1 + 2 \cos 2x + \cos^2 2x}{4} \\ &= \frac{1 + 2 \cos 2x + (1 + \cos 4x)/2}{4} = \frac{2 + 4 \cos 2x + 1 + \cos 4x}{8} \\ &= \frac{3 + 4 \cos 2x + \cos 4x}{8} = \frac{1}{8}(3 + 4 \cos 2x + \cos 4x) \end{aligned}$

$$24. \sin^4 x = (\sin^2 x)(\sin^2 x)$$

$$= \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 - \cos 2x}{2} \right)$$

$$= \frac{1 - 2 \cos 2x + \cos^2 2x}{4}$$

$$= \frac{1 - 2 \cos 2x + \left(\frac{1 + \cos 4x}{2} \right)}{4}$$

$$= \frac{2 - 4 \cos 2x + 1 + \cos 4x}{8}$$

$$= \frac{1}{8}(3 - 4 \cos 2x + \cos 4x)$$

$$25. (\sin^2 x)(\cos^2 x) = \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right)$$

$$= \frac{1 - \cos^2 2x}{4}$$

$$= \frac{1}{4} \left(1 - \frac{1 + \cos 4x}{2} \right)$$

$$= \frac{1}{8}(2 - 1 - \cos 4x)$$

$$= \frac{1}{8}(1 - \cos 4x)$$

$$26. \cos^6 x = (\cos^2 x)^3 = \left(\frac{1 + \cos 2x}{2} \right)^3$$

$$= \frac{1}{8}[1 + 3 \cos 2x + 3 \cos^2 2x + \cos^3 2x]$$

$$= \frac{1}{8} \left[1 + 3 \cos 2x + 3 \cdot \frac{1 + \cos 4x}{2} + \cos 2x \left(\frac{1 + \cos 4x}{2} \right) \right]$$

$$= \frac{1}{8} \left[\frac{5}{2} + 3 \cos 2x + \frac{3}{2} \cos 4x + \frac{1}{2} \cos 2x + \frac{1}{2} \cos 2x \cdot \cos 4x \right]$$

$$= \frac{1}{8} \left[\frac{5}{2} + \frac{7}{2} \cos 2x + \frac{3}{2} \cos 4x + \frac{1}{2} \frac{1}{2} (\cos 2x + \cos 6x) \right]$$

$$= \frac{1}{32}[10 + 15 \cos 2x + 6 \cos 4x + \cos 6x]$$

$$27. \sin^2 x \cos^4 x = \sin^2 x \cos^2 x \cos^2 x = \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right)$$

$$= \frac{1}{8}(1 - \cos 2x)(1 + \cos 2x)(1 + \cos 2x)$$

$$= \frac{1}{8}(1 - \cos^2 2x)(1 + \cos 2x)$$

$$= \frac{1}{8}(1 + \cos 2x - \cos^2 2x - \cos^3 2x)$$

$$= \frac{1}{8} \left[1 + \cos 2x - \left(\frac{1 + \cos 4x}{2} \right) - \cos 2x \left(\frac{1 + \cos 4x}{2} \right) \right]$$

$$= \frac{1}{16}[2 + 2 \cos 2x - 1 - \cos 4x - \cos 2x - \cos 2x \cos 4x]$$

$$= \frac{1}{16} \left[1 + \cos 2x - \cos 4x - \left(\frac{1}{2} \cos 2x + \frac{1}{2} \cos 6x \right) \right]$$

$$= \frac{1}{32}(2 + 2 \cos 2x - 2 \cos 4x - \cos 2x - \cos 6x)$$

$$= \frac{1}{32}(2 + \cos 2x - 2 \cos 4x - \cos 6x)$$

$$\begin{aligned}
28. \sin^4 x \cos^2 x &= \sin^2 x \sin^2 x \cos^2 x \\
&= \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 + \cos 2x}{2}\right) \\
&= \frac{1}{8}(1 - \cos 2x)(1 - \cos^2 2x) \\
&= \frac{1}{8}(1 - \cos 2x - \cos^2 2x + \cos^3 2x) \\
&= \frac{1}{8} \left[1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right) + \cos 2x \left(\frac{1 + \cos 4x}{2}\right) \right] \\
&= \frac{1}{16} [2 - 2 \cos 2x - 1 - \cos 4x + \cos 2x + \cos 2x \cos 4x] \\
&= \frac{1}{16} \left[1 - \cos 2x - \cos 4x + \frac{1}{2} \cos 2x + \frac{1}{2} \cos 6x \right] \\
&= \frac{1}{32} [2 - 2 \cos 2x - 2 \cos 4x + \cos 2x + \cos 6x] \\
&= \frac{1}{32} [2 - \cos 2x - 2 \cos 4x + \cos 6x]
\end{aligned}$$

$$\begin{aligned}
29. \sin^2 2x &= \frac{1 - \cos 4x}{2} \\
&= \frac{1}{2} - \frac{1}{2} \cos 4x \\
&= \frac{1}{2}(1 - \cos 4x)
\end{aligned}$$

$$\begin{aligned}
30. \cos^2 2x &= \frac{1 + \cos 4x}{2} \\
&= \frac{1}{2} + \frac{1}{2} \cos 4x \\
&= \frac{1}{2}(1 + \cos 4x)
\end{aligned}$$

$$\begin{aligned}
31. \cos^2 \frac{x}{2} &= \frac{1 + \cos x}{2} \\
&= \frac{1}{2} + \frac{1}{2} \cos x \\
&= \frac{1}{2}(1 + \cos x)
\end{aligned}$$

$$\begin{aligned}
32. \sin^2 \frac{x}{2} &= \frac{1 - \cos x}{2} \\
&= \frac{1}{2} - \frac{1}{2} \cos x \\
&= \frac{1}{2}(1 - \cos x)
\end{aligned}$$

$$\begin{aligned}
33. \sin^2 2x \cos^2 2x &= \left(\frac{1 - \cos 4x}{2}\right) \left(\frac{1 + \cos 4x}{2}\right) \\
&= \frac{1}{4}(1 - \cos 4x)(1 + \cos 4x) \\
&= \frac{1}{4}(1 - \cos^2 4x) \\
&= \frac{1}{4} \left(1 - \frac{1 + \cos 8x}{2} \right) \\
&= \frac{1 - \cos 8x}{8}
\end{aligned}$$

$$\begin{aligned}
34. \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} &= \left(\frac{1 - \cos x}{2}\right) \left(\frac{1 + \cos x}{2}\right) \\
&= \frac{1}{4}(1 - \cos x)(1 + \cos x) \\
&= \frac{1}{4}(1 - \cos^2 x) \\
&= \frac{1}{4} \left(1 - \frac{1 + \cos 2x}{2} \right) \\
&= \frac{1}{8}(1 - \cos 2x)
\end{aligned}$$

$$\begin{aligned}
 35. \sin^4 \frac{x}{2} &= \left(\sin^2 \frac{x}{2} \right) \left(\sin^2 \frac{x}{2} \right) \\
 &= \left(\frac{1 - \cos x}{2} \right) \left(\frac{1 - \cos x}{2} \right) \\
 &= \frac{1}{4} [1 - 2 \cos x + \cos^2 x] \\
 &= \frac{1}{4} \left[1 - 2 \cos x + \frac{1 + \cos 2x}{2} \right] \\
 &= \frac{1}{8} [2 - 4 \cos x + 1 + \cos 2x] \\
 &= \frac{1}{8} [3 - 4 \cos x + \cos 2x]
 \end{aligned}$$

$$\begin{aligned}
 36. \cos^4 \frac{x}{2} &= \left(\cos^2 \frac{x}{2} \right) \left(\cos^2 \frac{x}{2} \right) \\
 &= \left(\frac{1 + \cos x}{2} \right) \left(\frac{1 + \cos x}{2} \right) \\
 &= \frac{1}{4} [1 + 2 \cos x + \cos^2 x] \\
 &= \frac{1}{4} \left[1 + 2 \cos x + \frac{1 + \cos 2x}{2} \right] \\
 &= \frac{1}{8} [2 + 4 \cos x + 1 + \cos 2x] \\
 &= \frac{1}{8} [3 + 4 \cos x + \cos 2x]
 \end{aligned}$$

$$37. (a) \cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + (15/17)}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$

$$(b) \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - (15/17)}{2}} = \frac{\sqrt{17}}{17}$$

$$(c) \tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta} = \frac{8/17}{1 + (15/17)} = \frac{8}{32} = \frac{1}{4}$$

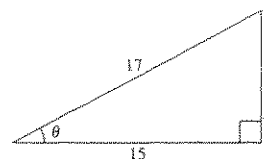
$$(d) \sec \frac{\theta}{2} = \frac{1}{\cos(\theta/2)} = \frac{1}{\sqrt{(1 + \cos \theta)/2}} = \frac{\sqrt{2}}{\sqrt{1 + (15/17)}} = \frac{\sqrt{17}}{4}$$

$$(e) \csc \frac{\theta}{2} = \frac{1}{\sin(\theta/2)} = \frac{1}{\sqrt{(1 - \cos \theta)/2}} = \frac{1}{\sqrt{[1 - (15/17)]/2}} = \frac{1}{1/\sqrt{17}} = \sqrt{17}$$

$$(f) \cot \frac{\theta}{2} = \frac{1 + \cos \theta}{\sin \theta} = \frac{1 + (15/17)}{8/17} = 4$$

$$(g) 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 2 \left(\frac{1}{\sqrt{17}} \right) \left(\frac{4\sqrt{17}}{17} \right) = \frac{8}{17}, \quad (= \sin \theta)$$

$$(h) 2 \cos \frac{\theta}{2} \tan \frac{\theta}{2} = 2 \sin \frac{\theta}{2} = \frac{2\sqrt{17}}{17}$$



$$\sin \theta = \frac{8}{17}$$

$$\cos \theta = \frac{15}{17}$$

$$38. (a) \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - (7/25)}{2}} = \sqrt{\frac{18}{50}} = \frac{3}{5}$$

$$(b) \cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + (7/25)}{2}} = \frac{4}{5}$$

$$(c) \tan \frac{\theta}{2} = \frac{\sin(\theta/2)}{\cos(\theta/2)} = \frac{3}{4}$$

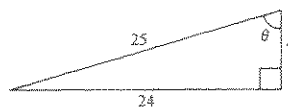
$$(d) \cot \frac{\theta}{2} = \frac{1}{\tan(\theta/2)} = \frac{4}{3}$$

$$(e) \sec \frac{\theta}{2} = \frac{1}{\cos(\theta/2)} = \frac{5}{4}$$

$$(f) \csc \frac{\theta}{2} = \frac{1}{\sin(\theta/2)} = \frac{5}{3}$$

$$(g) 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \sin \theta = \frac{24}{25}$$

$$(h) \cos 2\theta = 2 \cos^2 \theta - 1 = 2 \left(\frac{7}{25} \right)^2 - 1 = -\frac{527}{625}$$



$$39. \sin 15^\circ = \sin\left(\frac{1}{2} \cdot 30^\circ\right) = \sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

$$\cos 15^\circ = \cos\left(\frac{1}{2} \cdot 30^\circ\right) = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$$

$$\tan 15^\circ = \tan\left(\frac{1}{2} \cdot 30^\circ\right) = \frac{\sin 30^\circ}{1 + \cos 30^\circ} = \frac{1/2}{1 + (\sqrt{3}/2)} = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$$

$$40. \sin 165^\circ = \sin\left(\frac{1}{2} \cdot 330^\circ\right) = \sqrt{\frac{1 - \cos 330^\circ}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

$$\cos 165^\circ = \cos\left(\frac{1}{2} \cdot 330^\circ\right) = -\sqrt{\frac{1 + \cos 330^\circ}{2}} = -\sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = -\frac{1}{2}\sqrt{2 + \sqrt{3}}$$

$$\tan 165^\circ = \tan\left(\frac{1}{2} \cdot 330^\circ\right) = \frac{\sin 330^\circ}{1 + \cos 330^\circ} = \frac{-1/2}{1 + (\sqrt{3}/2)} = \frac{-1}{2 + \sqrt{3}} = \sqrt{3} - 2$$

$$41. \sin 112^\circ 30' = \sin\left(\frac{1}{2} \cdot 225^\circ\right) = \sqrt{\frac{1 - \cos 225^\circ}{2}} = \sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$$

$$\cos 112^\circ 30' = \cos\left(\frac{1}{2} \cdot 225^\circ\right) = -\sqrt{\frac{1 + \cos 225^\circ}{2}} = -\sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = -\frac{1}{2}\sqrt{2 - \sqrt{2}}$$

$$\tan 112^\circ 30' = \tan\left(\frac{1}{2} \cdot 225^\circ\right) = \frac{\sin 225^\circ}{1 + \cos 225^\circ} = \frac{-\sqrt{2}/2}{1 - (\sqrt{2}/2)} = -1 - \sqrt{2}$$

$$42. 157^\circ 30' = 157.5^\circ = \frac{1}{2}(315^\circ), \quad \text{Quadrant II}$$

$$\sin(157^\circ 30') = \sin\left(\frac{1}{2} \cdot 315^\circ\right) = \sqrt{\frac{1 - \cos 315^\circ}{2}} = \sqrt{\frac{1 - \sqrt{2}/2}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\cos(157^\circ 30') = \cos\left(\frac{1}{2} \cdot 315^\circ\right) = -\sqrt{\frac{1 + \cos 315^\circ}{2}} = -\sqrt{\frac{1 + \sqrt{2}/2}{2}} = -\frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\tan(157^\circ 30') = \tan\left(\frac{1}{2} \cdot 315^\circ\right) = \frac{\sin 315^\circ}{1 + \cos 315^\circ} = \frac{-\sqrt{2}/2}{1 + \sqrt{2}/2} = \frac{-\sqrt{2}}{2 + \sqrt{2}} = 1 - \sqrt{2}$$

$$43. \sin \frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 - \cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}}$$

$$\cos \frac{\pi}{8} = \cos\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \sqrt{\frac{1 + \cos(\pi/4)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$$

$$\tan \frac{\pi}{8} = \tan\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \frac{\sin(\pi/4)}{1 + \cos(\pi/4)} = \frac{\sqrt{2}/2}{1 + (\sqrt{2}/2)} = \sqrt{2} - 1$$

$$44. \sin \frac{\pi}{12} = \sin\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 - \cos(\pi/6)}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{3}}$$

$$\cos \frac{\pi}{12} = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \sqrt{\frac{1 + \cos(\pi/6)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$$

$$\tan \frac{\pi}{12} = \tan\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right] = \frac{\sin(\pi/6)}{1 + \cos(\pi/6)} = \frac{1/2}{1 + (\sqrt{3}/2)} = 2 - \sqrt{3}$$

$$\begin{aligned}
 45. \quad \sin \frac{3\pi}{8} &= \sin\left(\frac{1}{2} \cdot \frac{3\pi}{4}\right) = \sqrt{\frac{1 - \cos(3\pi/4)}{2}} = \sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2 + \sqrt{2}} \\
 \cos \frac{3\pi}{8} &= \cos\left(\frac{1}{2} \cdot \frac{3\pi}{4}\right) = \sqrt{\frac{1 + \cos(3\pi/4)}{2}} = \sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = \frac{1}{2}\sqrt{2 - \sqrt{2}} \\
 \tan \frac{3\pi}{8} &= \tan\left(\frac{1}{2} \cdot \frac{3\pi}{4}\right) = \frac{\sin(3\pi/4)}{1 + \cos(3\pi/4)} = \frac{\sqrt{2}/2}{1 - (\sqrt{2}/2)} = \frac{\sqrt{2}}{2 - \sqrt{2}} = \sqrt{2} + 1
 \end{aligned}$$

$$46. \quad \frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right), \text{ Quadrant II}$$

$$\begin{aligned}
 \sin\left(\frac{7\pi}{12}\right) &= \sin\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = \sqrt{\frac{1 - \cos(7\pi/6)}{2}} = \sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2} \\
 \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = -\sqrt{\frac{1 + \cos(7\pi/6)}{2}} = -\sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = -\frac{\sqrt{2 - \sqrt{3}}}{2} \\
 \tan\left(\frac{7\pi}{12}\right) &= \tan\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right) = \frac{\sin(7\pi/6)}{1 + \cos(7\pi/6)} = \frac{-(1/2)}{1 - (\sqrt{3}/2)} = \frac{1}{\sqrt{3} - 2} = -2 - \sqrt{3}
 \end{aligned}$$

$$47. \quad \sin u = \frac{5}{13}, \quad \frac{\pi}{2} < u < \pi \Rightarrow \cos u = -\frac{12}{13}$$

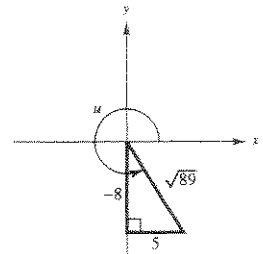
$$\begin{aligned}
 \sin\left(\frac{u}{2}\right) &= \sqrt{\frac{1 - \cos u}{2}} \\
 &= \sqrt{\frac{1 + (12/13)}{2}} = \frac{5\sqrt{26}}{26} \\
 \cos\left(\frac{u}{2}\right) &= \sqrt{\frac{1 + \cos u}{2}} \\
 &= \sqrt{\frac{1 - (12/13)}{2}} = \frac{\sqrt{26}}{26} \\
 \tan\left(\frac{u}{2}\right) &= \frac{\sin u}{1 + \cos u} = \frac{5/13}{1 - (12/13)} = \frac{5}{1} = 5
 \end{aligned}$$

$$48. \quad \cos u = \frac{7}{25}, \quad 0 < u < \frac{\pi}{2}, \text{ Quadrant I}$$

$$\begin{aligned}
 \sin u &= \sqrt{1 - \cos^2 u} = \sqrt{1 - \frac{49}{625}} = \frac{24}{25} \\
 \sin \frac{u}{2} &= \sqrt{\frac{1 - \cos u}{2}} \\
 &= \sqrt{\frac{1 - (7/25)}{2}} = \sqrt{\frac{9}{25}} = 0.6 = \frac{3}{5} \\
 \cos \frac{u}{2} &= \sqrt{\frac{1 + \cos u}{2}} \\
 &= \sqrt{\frac{1 + (7/25)}{2}} = \sqrt{\frac{16}{25}} = 0.8 = \frac{4}{5} \\
 \tan \frac{u}{2} &= \frac{1 - \cos u}{\sin u} = \frac{1 - (7/25)}{24/25} = \frac{18}{24} = \frac{3}{4}
 \end{aligned}$$

$$49. \quad \tan u = -\frac{8}{5}, \quad \frac{3\pi}{2} < u < 2\pi, \text{ Quadrant IV}$$

$$\begin{aligned}
 \sin u &= -\frac{8}{\sqrt{89}}, \quad \cos u = \frac{5}{\sqrt{89}} \\
 \sin\left(\frac{u}{2}\right) &= \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 - (5/\sqrt{89})}{2}} = \sqrt{\frac{\sqrt{89} - 5}{2\sqrt{89}}} = \sqrt{\frac{89 - 5\sqrt{89}}{178}} \\
 \cos\left(\frac{u}{2}\right) &= -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 + (5/\sqrt{89})}{2}} = -\sqrt{\frac{\sqrt{89} + 5}{2\sqrt{89}}} = -\sqrt{\frac{89 + 5\sqrt{89}}{178}} \\
 \tan\left(\frac{u}{2}\right) &= \frac{1 - \cos u}{\sin u} = \frac{1 - (5/\sqrt{89})}{-8/\sqrt{89}} = \frac{5 - \sqrt{89}}{8}
 \end{aligned}$$

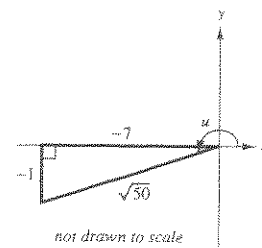


50. $\cot u = 7, \pi < u < \frac{3\pi}{2}$, Quadrant III

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (7/\sqrt{50})}{2}} = \sqrt{\frac{\sqrt{50} + 7}{2\sqrt{50}}} = \frac{\sqrt{50 + 7\sqrt{50}}}{10}$$

$$\cos \frac{u}{2} = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - 7/\sqrt{50}}{2}} = -\sqrt{\frac{\sqrt{50} - 7}{2\sqrt{50}}} = -\frac{\sqrt{50 - 7\sqrt{50}}}{10}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 + 7/\sqrt{50}}{-1/\sqrt{50}} = -(\sqrt{50} + 7)$$



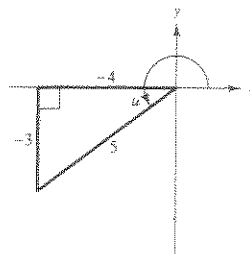
51. $\csc u = -\frac{5}{3}, \pi < u < \frac{3\pi}{2}$, Quadrant III

$$\sin u = -\frac{3}{5}, \cos u = -\frac{4}{5}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (4/5)}{2}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos\left(\frac{u}{2}\right) = -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - (4/5)}{2}} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + (4/5)}{-3/5} = -3$$



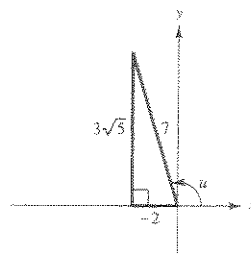
52. $\sec u = \frac{-7}{2}, \frac{\pi}{2} < u < \pi$

$$\cos u = \frac{-2}{7}, \sin u = \frac{3\sqrt{5}}{7}$$

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (2/7)}{2}} = \sqrt{\frac{9}{14}} = \frac{3\sqrt{14}}{14}$$

$$\cos \frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (2/7)}{2}} = \sqrt{\frac{5}{14}} = \frac{\sqrt{70}}{14}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1 + (2/7)}{3(\sqrt{5}/7)} = \frac{9}{3\sqrt{5}} = \frac{3\sqrt{5}}{5}$$



53. $\sqrt{\frac{1 - \cos 6x}{2}} = |\sin 3x|$

54. $\sqrt{\frac{1 + \cos 4x}{2}} = \left|\cos \frac{4x}{2}\right| = |\cos 2x|$

55. $-\sqrt{\frac{1 - \cos 8x}{1 + \cos 8x}} = -\frac{\sqrt{(1 - \cos 8x)/2}}{\sqrt{(1 + \cos 8x)/2}}$

$$= -\left|\frac{\sin 4x}{\cos 4x}\right| = -|\tan 4x|$$

56. $-\sqrt{\frac{1 - \cos(x-1)}{2}} = -\left|\sin\left(\frac{x-1}{2}\right)\right|$

$$57. \quad \sin \frac{x}{2} - \cos x = 0$$

$$\pm \sqrt{\frac{1 - \cos x}{2}} = \cos x$$

$$\frac{1 - \cos x}{2} = \cos^2 x$$

$$0 = 2 \cos^2 x + \cos x - 1$$

$$= (2 \cos x - 1)(\cos x + 1)$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

By checking these values in the original equations, we see that $x = \pi/3$ and $x = 5\pi/3$ are the only solutions. $x = \pi$ is extraneous.

$$58. \quad h(x) = \sin \frac{x}{2} + \cos x - 1$$

$$\sin \frac{x}{2} + \cos x - 1 = 0$$

$$\pm \sqrt{\frac{1 - \cos x}{2}} = 1 - \cos x$$

$$\frac{1 - \cos x}{2} = 1 - 2 \cos x + \cos^2 x$$

$$1 - \cos x = 2 - 4 \cos x + 2 \cos^2 x$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$2 \cos x - 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = 1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = 0$$

$$59. \quad \cos \frac{x}{2} - \sin x = 0$$

$$\pm \sqrt{\frac{1 + \cos x}{2}} = \sin x$$

$$\frac{1 + \cos x}{2} = \sin^2 x$$

$$1 + \cos x = 2 \sin^2 x$$

$$1 + \cos x = 2 - 2 \cos^2 x$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$2 \cos x - 1 = 0 \quad \text{or} \quad \cos x + 1 = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$\pi/3$, π , and $5\pi/3$ are all solutions to the equation.

$$60. \quad g(x) = \tan \frac{x}{2} - \sin x$$

$$\tan \frac{x}{2} - \sin x = 0$$

$$\frac{1 - \cos x}{\sin x} = \sin x$$

$$1 - \cos x = \sin^2 x$$

$$1 - \cos x = 1 - \cos^2 x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \cos x = 1$$

$$x = 0$$

0, $\pi/2$, and $3\pi/2$ are all solutions to the equation.

$$61. \quad 6 \sin \frac{\pi}{3} \cos \frac{\pi}{3} = 6 \cdot \frac{1}{2} \left[\sin \left(\frac{\pi}{3} + \frac{\pi}{3} \right) + \sin \left(\frac{\pi}{3} - \frac{\pi}{3} \right) \right] = 3 \left[\sin \frac{2\pi}{3} + \sin 0 \right] = 3 \sin \frac{2\pi}{3}$$

$$62. \quad 4 \sin \frac{\pi}{3} \cos \frac{5\pi}{6} = 4 \cdot \frac{1}{2} \left[\sin \left(\frac{\pi}{3} + \frac{5\pi}{6} \right) + \sin \left(\frac{\pi}{3} - \frac{5\pi}{6} \right) \right] = 2 \left[\sin \frac{7\pi}{6} + \sin \left(-\frac{\pi}{2} \right) \right] = 2 \left(\sin \frac{7\pi}{6} - \sin \frac{\pi}{2} \right)$$

$$63. \sin 5\theta \cos 3\theta = \frac{1}{2}[\sin(5\theta + 3\theta) + \sin(5\theta - 3\theta)] = \frac{1}{2}(\sin 8\theta + \sin 2\theta)$$

$$64. 5 \sin 3\alpha \sin 4\alpha = 5 \cdot \frac{1}{2}[\cos(3\alpha - 4\alpha) - \cos(3\alpha + 4\alpha)] = \frac{5}{2}[\cos(-\alpha) - \cos(7\alpha)] = \frac{5}{2}[\cos \alpha - \cos 7\alpha]$$

$$65. 10 \cos 75^\circ \cos 15^\circ = 5[\cos(75^\circ - 15^\circ) + \cos(75^\circ + 15^\circ)] = 5[\cos 60^\circ + \cos 90^\circ]$$

$$66. 6 \sin 45^\circ \cos 15^\circ = 3[\sin(45^\circ + 15^\circ) + \sin(45^\circ - 15^\circ)] = 3[\sin 60^\circ + \sin 30^\circ]$$

$$67. 5 \cos(-5\beta) \cos 3\beta = 5 \cdot \frac{1}{2}[\cos(-5\beta - 3\beta) + \cos(-5\beta + 3\beta)] \\ = \frac{5}{2}[\cos(-8\beta) + \cos(-2\beta)] = \frac{5}{2}(\cos 8\beta + \cos 2\beta)$$

$$68. \cos 2\theta \cos 4\theta = \frac{1}{2}[\cos(2\theta - 4\theta) + \cos(2\theta + 4\theta)] = \frac{1}{2}[\cos(-2\theta) + \cos 6\theta] = \frac{1}{2}(\cos 2\theta + \cos 6\theta)$$

$$69. \sin(x + y) \sin(x - y) = \frac{1}{2}[\cos((x + y) - (x - y)) - \cos((x + y) + (x - y))] = \frac{1}{2}[\cos 2y - \cos 2x]$$

$$70. \sin(x + y) \cos(x - y) = \frac{1}{2}[\sin((x + y) + (x - y)) + \sin((x + y) - (x - y))] = \frac{1}{2}[\sin 2x + \sin 2y]$$

$$71. \cos(\theta - \pi) \sin(\theta + \pi) = \frac{1}{2}[\sin((\theta - \pi) + (\theta + \pi)) - \sin((\theta - \pi) - (\theta + \pi))] \\ = \frac{1}{2}[\sin 2\theta - \sin(-2\pi)] = \frac{1}{2}[\sin 2\theta + \sin 2\pi]$$

$$72. \sin(\theta + \pi) \sin(\theta - \pi) = \frac{1}{2}[\cos((\theta + \pi) - (\theta - \pi)) - \cos((\theta + \pi) + (\theta - \pi))] = \frac{1}{2}[\cos 2\pi - \cos 2\theta]$$

$$73. \sin 5\theta - \sin \theta = 2 \cos\left(\frac{5\theta + \theta}{2}\right) \sin\left(\frac{5\theta - \theta}{2}\right) \qquad 74. \sin 3\theta + \sin \theta = 2 \sin\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right) \\ = 2 \cos 3\theta \cdot \sin 2\theta \qquad \qquad \qquad = 2 \sin 2\theta \cos \theta$$

$$75. \cos 6x + \cos 2x = 2 \cos\left(\frac{6x + 2x}{2}\right) \cos\left(\frac{6x - 2x}{2}\right) \qquad 76. \sin x + \sin 7x = 2 \sin\left(\frac{x + 7x}{2}\right) \cos\left(\frac{x - 7x}{2}\right) \\ = 2 \cos 4x \cos 2x \qquad \qquad \qquad = 2 \sin 4x \cos(-3x) \\ = 2 \sin 4x \cos 3x$$

$$77. \sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos\left(\frac{\alpha + \beta + \alpha - \beta}{2}\right) \sin\left(\frac{\alpha + \beta - \alpha + \beta}{2}\right) = 2 \cos \alpha \sin \beta$$

$$78. \cos(\phi + 2\pi) + \cos \phi = 2 \cos\left(\frac{\phi + 2\pi + \phi}{2}\right) \cos\left(\frac{\phi + 2\pi - \phi}{2}\right) = 2 \cos(\phi + \pi) \cos \pi = -2 \cos(\phi + \pi)$$

$$79. \cos\left(\theta + \frac{\pi}{2}\right) - \cos\left(\theta - \frac{\pi}{2}\right) = -2 \sin\left(\frac{\theta + (\pi/2) + \theta - (\pi/2)}{2}\right) \sin\left(\frac{\theta + (\pi/2) - \theta + (\pi/2)}{2}\right) \\ = -2 \sin \theta \sin \frac{\pi}{2} = -2 \sin \theta$$

$$80. \sin\left(x + \frac{\pi}{2}\right) + \sin\left(x - \frac{\pi}{2}\right) = 2 \sin\left(\frac{x + (\pi/2) + x - (\pi/2)}{2}\right) \cos\left(\frac{x + (\pi/2) - x + (\pi/2)}{2}\right) = 2 \sin x \cos \frac{\pi}{2} = 0$$

$$81. \sin 195^\circ + \sin 105^\circ = 2 \sin\left(\frac{195^\circ + 105^\circ}{2}\right) \cos\left(\frac{195^\circ - 105^\circ}{2}\right) = 2 \sin(150^\circ) \cos(45^\circ) = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

$$82. \cos 165^\circ - \cos 75^\circ = -2 \sin\left(\frac{165^\circ + 75^\circ}{2}\right) \sin\left(\frac{165^\circ - 75^\circ}{2}\right) = -2 \sin(120^\circ) \sin(45^\circ) \\ = -2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}}{2}$$

$$83. \cos \frac{5\pi}{12} + \cos \frac{\pi}{12} = 2 \cos\left(\frac{(5\pi/12) + (\pi/12)}{2}\right) \cos\left(\frac{(5\pi/12) - (\pi/12)}{2}\right) \\ = 2 \cos\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$$

$$84. \sin \frac{11\pi}{12} - \sin \frac{7\pi}{12} = 2 \cos\left(\frac{(11\pi/12) + (7\pi/12)}{2}\right) \sin\left(\frac{(11\pi/12) - (7\pi/12)}{2}\right) \\ = 2 \cos\left(\frac{3\pi}{4}\right) \sin\left(\frac{\pi}{6}\right) = 2\left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{2}}{2}$$

$$85. \quad \sin 6x + \sin 2x = 0$$

$$2 \sin\left(\frac{6x + 2x}{2}\right) \cos\left(\frac{6x - 2x}{2}\right) = 0$$

$$\sin 4x \cos 2x = 0$$

$$\sin 4x = 0 \quad \text{or} \quad \cos 2x = 0$$

$$4x = n\pi \quad 2x = \frac{\pi}{2} + n\pi$$

$$x = \frac{n\pi}{4} \quad x = \frac{\pi}{4} + \frac{n\pi}{2}$$

In the interval we have $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$.

$$86. \quad h(x) = \cos 2x - \cos 6x$$

$$\cos 2x - \cos 6x = 0$$

$$-2 \sin 4x \sin(-2x) = 0$$

$$2 \sin 4x \sin 2x = 0$$

$$\sin 4x = 0 \quad \text{OR} \quad \sin 2x = 0$$

$$4x = n\pi \quad 2x = n\pi$$

$$x = \frac{n\pi}{4} \quad x = \frac{n\pi}{2}$$

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$$

$$87. \frac{\cos 2x}{\sin 3x - \sin x} - 1 = 0$$

$$\frac{\cos 2x}{\sin 3x - \sin x} = 1$$

$$\frac{\cos 2x}{2 \cos 2x \sin x} = 1$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

88.

$$f(x) = \sin^2 3x - \sin^2 x$$

$$\sin^2 3x - \sin^2 x = 0$$

$$(\sin 3x + \sin x)(\sin 3x - \sin x) = 0$$

$$(2 \sin 2x \cos x)(2 \cos 2x \sin x) = 0$$

$$\sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \text{ or}$$

$$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or}$$

$$\cos 2x = 0 \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \text{ or}$$

$$\sin x = 0 \Rightarrow x = 0, \pi$$

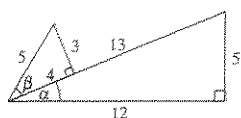


Figure for Exercises 89-92

$$89. \sin^2 \alpha = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$$

$$\sin^2 \alpha = 1 - \cos^2 \alpha = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$$

$$90. \cos^2 \alpha = (\cos \alpha)^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$$

$$\cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

$$91. \sin \alpha \cos \beta = \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13}$$

$$\begin{aligned} \sin \alpha \cos \beta &= \cos\left(\frac{\pi}{2} - \alpha\right) \sin\left(\frac{\pi}{2} - \beta\right) \\ &= \left(\frac{5}{13}\right)\left(\frac{4}{5}\right) = \frac{4}{13} \end{aligned}$$

$$92. \cos \alpha \sin \beta = \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65}$$

$$\begin{aligned} \cos \alpha \sin \beta &= \sin\left(\frac{\pi}{2} - \alpha\right) \cos\left(\frac{\pi}{2} - \beta\right) \\ &= \left(\frac{12}{13}\right)\left(\frac{3}{5}\right) = \frac{36}{65} \end{aligned}$$

$$93. \csc 2\theta = \frac{1}{\sin 2\theta}$$

$$= \frac{1}{2 \sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta} \cdot \frac{1}{2 \cos \theta}$$

$$= \frac{\csc \theta}{2 \cos \theta}$$

$$94. \sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$= \frac{1/\cos^2 \theta}{1 - (\sin^2 \theta / \cos^2 \theta)}$$

$$= \frac{\sec^2 \theta}{1 - \tan^2 \theta}$$

$$= \frac{\sec^2 \theta}{1 - (\sec^2 \theta - 1)}$$

$$= \frac{\sec^2 \theta}{2 - \sec^2 \theta}$$

$$\begin{aligned} 95. \cos^2 2\alpha - \sin^2 2\alpha &= \cos[2(2\alpha)] \\ &= \cos 4\alpha \end{aligned}$$

$$\begin{aligned} 96. \cos^4 x - \sin^4 x &= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\ &= (\cos 2x)(1) = \cos 2x \end{aligned}$$

$$\begin{aligned} 97. (\sin x + \cos x)^2 &= \sin^2 x + 2 \sin x \cos x + \cos^2 x \\ &= (\sin^2 x + \cos^2 x) + 2 \sin x \cos x = 1 + \sin 2x \end{aligned}$$

$$\begin{aligned}
 98. \sin \frac{\alpha}{3} \cos \frac{\alpha}{3} &= \frac{1}{2} \left[\sin \left(\frac{\alpha}{3} + \frac{\alpha}{3} \right) + \sin \left(\frac{\alpha}{3} - \frac{\alpha}{3} \right) \right] \\
 &= \frac{1}{2} \sin \frac{2\alpha}{3}
 \end{aligned}$$

$$\begin{aligned}
 99. 1 + \cos 10y &= 1 + \cos^2 5y - \sin^2 5y \\
 &= 1 + \cos^2 5y - (1 - \cos^2 5y) \\
 &= 2 \cos^2 5y
 \end{aligned}$$

$$\begin{aligned}
 100. \frac{\cos 3\beta}{\cos \beta} &= \frac{\cos(2\beta + \beta)}{\cos \beta} \\
 &= \frac{\cos 2\beta \cos \beta - \sin 2\beta \sin \beta}{\cos \beta} \\
 &= \frac{(1 - 2 \sin^2 \beta) \cos \beta - 2 \sin \beta \cos \beta \sin \beta}{\cos \beta} \\
 &= (1 - 2 \sin^2 \beta) - 2 \sin^2 \beta \\
 &= 1 - 4 \sin^2 \beta
 \end{aligned}$$

$$\begin{aligned}
 101. \sec \frac{u}{2} &= \frac{1}{\cos(u/2)} \\
 &= \pm \sqrt{\frac{2}{1 + \cos u}} \\
 &= \pm \sqrt{\frac{2 \sin u}{\sin u(1 + \cos u)}} \\
 &= \pm \sqrt{\frac{2 \sin u}{\sin u + \sin u \cos u}} \\
 &= \pm \sqrt{\frac{(2 \sin u)/(\cos u)}{(\sin u)/(\cos u) + (\sin u \cos u)/(\cos u)}} \\
 &= \pm \sqrt{\frac{2 \tan u}{\tan u + \sin u}}
 \end{aligned}$$

$$102. \tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{1}{\sin u} - \frac{\cos u}{\sin u} = \csc u - \cot u$$

$$\begin{aligned}
 103. \cos 3\beta &= \cos(2\beta + \beta) \\
 &= \cos 2\beta \cos \beta - \sin 2\beta \sin \beta = (\cos^2 \beta - \sin^2 \beta) \cos \beta - 2 \sin \beta \cos \beta \sin \beta \\
 &= \cos^3 \beta - \sin^2 \beta \cos \beta - 2 \sin^2 \beta \cos \beta = \cos^3 \beta - 3 \sin^2 \beta \cos \beta
 \end{aligned}$$

$$\begin{aligned}
 104. \sin 4\beta &= 2 \sin 2\beta \cos 2\beta \\
 &= 2[2 \sin \beta \cos \beta (\cos^2 \beta - \sin^2 \beta)] \\
 &= 2[2 \sin \beta \cos \beta (1 - \sin^2 \beta - \sin^2 \beta)] \\
 &= 4 \sin \beta \cos \beta (1 - 2 \sin^2 \beta)
 \end{aligned}$$

$$\begin{aligned}
 105. \frac{\cos 4x - \cos 2x}{2 \sin 3x} &= \frac{-2 \sin \left(\frac{4x + 2x}{2} \right) \sin \left(\frac{4x - 2x}{2} \right)}{2 \sin 3x} \\
 &= \frac{-2 \sin 3x \sin x}{2 \sin 3x} \\
 &= -\sin x
 \end{aligned}$$

$$\begin{aligned}
 106. \frac{\cos 3x - \cos x}{\sin 3x - \sin x} &= \frac{-2 \sin \left(\frac{3x + x}{2} \right) \sin \left(\frac{3x - x}{2} \right)}{2 \cos \left(\frac{3x + x}{2} \right) \sin \left(\frac{3x - x}{2} \right)} \\
 &= \frac{-2 \sin 2x \sin x}{2 \cos 2x \sin x} = -\tan 2x
 \end{aligned}$$

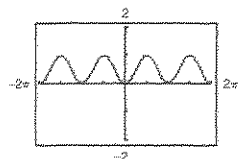
$$\begin{aligned}
 107. \frac{\cos 4x + \cos 2x}{\sin 4x + \sin 2x} &= \frac{2 \cos(3x) \cos x}{2 \sin(3x) \cos x} \\
 &= \cot 3x
 \end{aligned}$$

$$108. \frac{\cos t + \cos 3t}{\sin 3t - \sin t} = \frac{2 \cos 2t \cos(-t)}{2 \cos 2t \sin t} = \frac{\cos t}{\sin t} = \cot t$$

$$109. \sin \left(\frac{\pi}{6} + x \right) + \sin \left(\frac{\pi}{6} - x \right) = 2 \sin \left(\frac{\frac{\pi}{6} + x + \frac{\pi}{6} - x}{2} \right) \cos \left(\frac{\frac{\pi}{6} + x - \frac{\pi}{6} - x}{2} \right) = 2 \sin \frac{\pi}{6} \cos x = \cos x$$

$$110. \cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = 2 \cos\left(\frac{\frac{\pi}{3} + x + \frac{\pi}{3} - x}{2}\right) \cos\left(\frac{\frac{\pi}{3} + x - \frac{\pi}{3} + x}{2}\right) = 2 \cos \frac{\pi}{3} \cos x = \cos x$$

$$111. \sin^2 x = \frac{1 - \cos 2x}{2} = \frac{1}{2} - \frac{\cos 2x}{2}$$

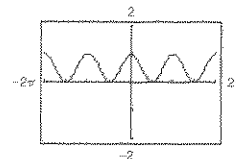


$$112. f(x) = \cos^2 x = \frac{1 + \cos 2x}{2} = \frac{1}{2} + \frac{\cos 2x}{2}$$

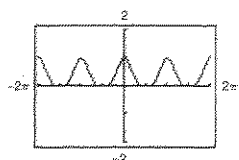
Shifted upward by $\frac{1}{2}$ unit.

$$\text{Amplitude: } |a| = \frac{1}{2}$$

$$\text{Period: } \frac{2\pi}{2} = \pi$$

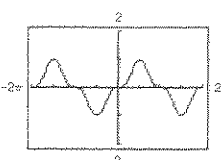


$$113. f(x) = \cos^4 x = \frac{1}{8}(3 + 4 \cos 2x + \cos 4x)$$

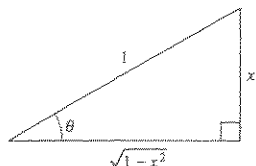


$$114. f(x) = \sin^3 x$$

$$= \sin x \left(\frac{1 - \cos 2x}{2} \right)$$

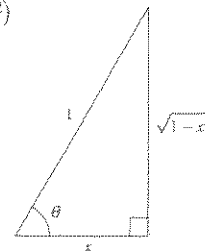


$$115. \sin(2 \arcsin x) = 2 \sin(\arcsin x) \cos(\arcsin x) \\ = 2x \sqrt{1 - x^2}$$



$$116. \text{Let } \theta = \arccos x.$$

$$\begin{aligned} \cos(2 \arccos x) &= \cos^2(\arccos x) - \sin^2(\arccos x) \\ &= x^2 - (1 - x^2) \\ &= 2x^2 - 1 \end{aligned}$$

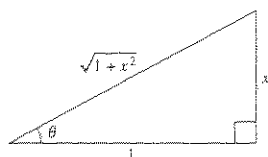


$$117. \cos(2 \arcsin x) = 1 - 2 \sin^2(\arcsin x) \\ = 1 - 2x^2$$

$$118. \text{Let } u = \arccos x.$$

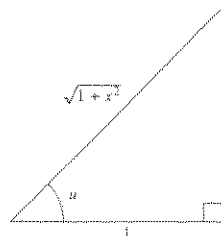
$$\begin{aligned} \sin(2 \arccos x) &= 2 \sin(\arccos x) \cos(\arccos x) \\ &= 2 \sqrt{1 - x^2}(x) = 2x \sqrt{1 - x^2} \end{aligned}$$

$$119. \cos(2 \arctan x) = 1 - 2 \sin^2(\arctan x) \\ = 1 - 2 \left(\frac{x}{\sqrt{1 + x^2}} \right)^2 \\ = 1 - \frac{2x^2}{1 + x^2} \\ = \frac{1 - x^2}{1 + x^2}$$

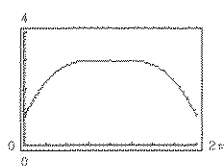


$$120. \text{Let } u = \arctan x.$$

$$\begin{aligned} \sin(2 \arctan x) &= 2 \sin(\arctan x) \cos(\arctan x) \\ &= 2 \frac{x}{\sqrt{1 + x^2}} \cdot \frac{1}{\sqrt{1 + x^2}} \\ &= \frac{2x}{1 + x^2} \end{aligned}$$



121. (a) $y = 4 \sin \frac{x}{2} + \cos x$


Maximum: $(\pi, 3)$

(b) $2 \cos \frac{x}{2} - \sin x = 0$

$$2 \left(\pm \sqrt{\frac{1 + \cos x}{2}} \right) = \sin x$$

$$4 \left(\frac{1 + \cos x}{2} \right) = \sin^2 x$$

$$2(1 + \cos x) = 1 - \cos^2 x$$

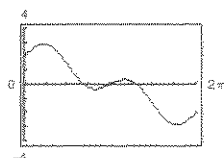
$$\cos^2 x + 2 \cos x + 1 = 0$$

$$(\cos x + 1)^2 = 0$$

$$\cos x = -1$$

$$x = \pi$$

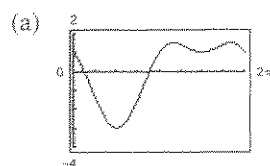
123. (a) $y = 2 \cos \frac{x}{2} + \sin 2x$


Maximum: $(0.699, 2.864)$

Minimum: $(5.584, -2.864)$

- (b) $2 \cos 2x - \sin(x/2) = 0$ has four zeros on $[0, 2\pi)$. Two of the zeros are $x \approx 0.699$ and $x \approx 5.584$. (The other two are $2.608, 3.675$.)

122. $f(x) = \cos 2x - 2 \sin x$


Maximum points: $(3.6652, 1.5), (5.7596, 1.5)$

Minimum point: $(1.5708, -3)$

(b) $-2 \cos x(2 \sin x + 1) = 0$

$$-2 \cos x = 0 \quad \text{or} \quad 2 \sin x + 1 = 0$$

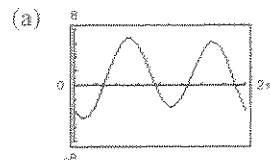
$$\cos x = 0 \quad \sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\frac{\pi}{2} \approx 1.5708 \quad \frac{7\pi}{6} \approx 3.6652$$

$$\frac{3\pi}{2} \approx 4.7124 \quad \frac{11\pi}{6} \approx 5.7596$$

124. $f(x) = 2 \sin \frac{x}{2} - 5 \cos \left(2x - \frac{\pi}{4} \right)$


Maximum point: $(1.9907, 6.6705)$

Minimum point: $(0.3434, -4.6340)$

(b) $10 \sin \left(2x - \frac{\pi}{4} \right) + \cos \frac{x}{2} = 0$

$$x \approx 0.343, 1.991, 3.544, 5.064$$

The first and second solutions correspond to the maximum and minimum points in part (a).

125. (a) $f(x) = \sin 2x - \sin x = 0$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x(2 \cos x - 1) = 0$$

$$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$$

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$$

(b) $2 \cos 2x - \cos x = 0$

$$2(2 \cos^2 x - 1) - \cos x = 0$$

$$4 \cos^2 x - \cos x - 2 = 0$$

$$\cos x = \frac{1 \pm \sqrt{1 + 32}}{8} = \frac{1 \pm \sqrt{33}}{8}$$

$$x = \arccos\left(\frac{1 \pm \sqrt{33}}{8}\right)$$

$$x = 2\pi - \arccos\left(\frac{1 \pm \sqrt{33}}{8}\right)$$

$$x \approx 0.5678, 2.2057, 4.0775, 5.7154$$

126. (a) $f(x) = \cos 2x + \sin x = 0$

$$1 - 2 \sin^2 x + \sin x = 0$$

$$2 \sin^2 x - \sin x - 1 = 0$$

$$(2 \sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\sin x = 1 \Rightarrow x = \frac{\pi}{2}$$

(b) $-2 \sin 2x + \cos x = 0$

$$-4 \sin x \cos x + \cos x = 0$$

$$\cos x(1 - 4 \sin x) = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \arcsin\left(\frac{1}{4}\right), \pi - \arcsin\left(\frac{1}{4}\right)$$

$$x \approx 1.5708, 4.7124, 0.2527, 2.8889$$

127. (a) $r = \frac{1}{32} v_0^2 \sin 2\theta$

$$= \frac{1}{32} v_0^2 (2 \sin \theta \cos \theta)$$

$$= \frac{1}{16} v_0^2 \sin \theta \cos \theta$$

(b) $r = \frac{1}{16} v_0^2 \sin \theta \cos \theta$

$$= \frac{1}{16} (80)^2 \sin 42^\circ \cos 42^\circ \approx 198.90 \text{ feet}$$

(c) $r = \frac{1}{16} v_0^2 \sin \theta \cos \theta$

$$300 = \frac{1}{16} v_0^2 \sin 40^\circ \cos 40^\circ$$

$$v_0^2 = \frac{300(16)}{\sin 40^\circ \cos 40^\circ} \approx 9748.0955$$

$$v_0 \approx 98.73 \text{ feet per second}$$

(d) $\sin 2\theta$ is greatest when $\theta = 45^\circ$.

128. (a) $\sin\left(\frac{\theta}{2}\right) = \frac{b/2}{10} \Rightarrow b = 20 \sin \frac{\theta}{2}$

$$\cos\left(\frac{\theta}{2}\right) = \frac{h}{10} \Rightarrow h = 10 \cos \frac{\theta}{2}$$

$$A = \frac{1}{2}bh = \frac{1}{2}\left(20 \sin \frac{\theta}{2}\right)\left(10 \cos \frac{\theta}{2}\right)$$

$$= 100 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

(b) $A = 50\left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}\right) = 50 \sin \theta$

The area is maximum when $\theta = \frac{\pi}{2}$, $A = 50$.

129. $\frac{x}{2} = 2r \sin^2 \frac{\theta}{2}$, $x = 4r \left[\sin \frac{\theta}{2} \right]^2 = 4r \frac{1 - \cos \theta}{2} = 2r(1 - \cos \theta)$

130. $\sin \frac{\theta}{2} = \frac{1}{M}$

(a) $\sin \frac{\theta}{2} = \frac{1}{1} = 1 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2} \Rightarrow \theta = \pi = 180^\circ$ (d) $\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \frac{1}{M}$

(b) $\sin \frac{\theta}{2} = \frac{1}{4.5} = \frac{2}{9}$ $\frac{1 - \cos \theta}{2} = \frac{1}{M^2}$

$\frac{\theta}{2} = \arcsin\left(\frac{2}{9}\right) \approx 0.2241$ $1 - \cos \theta = \frac{2}{M^2}$

$\theta \approx 0.4482 \approx 25.7^\circ$

$\cos \theta = 1 - \frac{2}{M^2}$

(c) $M = 1 \Rightarrow \text{Speed} = 760 \text{ mph}$

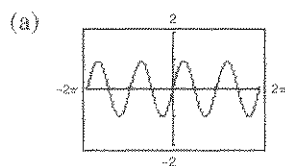
$M = 4.5 \Rightarrow \frac{\text{Speed}}{760} = 4.5 \Rightarrow \text{Speed} = 3420 \text{ mph}$

131. False. If $x = \pi$, $\sin \frac{x}{2} = \sin \frac{\pi}{2} = 1$, whereas

$-\sqrt{\frac{1 - \cos \pi}{2}} = -1.$

132. True. $\sin(\pi) = 0$ and $y = 4 - 8 \sin^2 \pi = 4$, a maximum.

133. $f(x) = 2 \sin x \left[2 \cos^2\left(\frac{x}{2}\right) - 1 \right]$



(b) The graph appears to be that of $y = \sin 2x$.

(c) $2 \sin x \left[2 \cos^2\left(\frac{x}{2}\right) - 1 \right] = 2 \sin x \left[2 \frac{1 + \cos x}{2} - 1 \right]$
 $= 2 \sin x [\cos x] = \sin 2x$

134. (a) $f(x) = \sin^4 x + \cos^4 x$. From Example 5 and Exercise 23,

$f(x) = \frac{1}{8}(3 - 4 \cos 2x + \cos 4x) + \frac{1}{8}(3 + 4 \cos 2x + \cos 4x) = \frac{3}{4} + \frac{1}{4} \cos 4x$

(b) Sample answer:

$f(x) = \sin^2 x \cdot \sin^2 x + \cos^2 x \cdot \cos^2 x = \left(\frac{1 - \cos 2x}{2} \right)^2 + \left(\frac{1 + \cos 2x}{2} \right)^2$
 $= \frac{1}{4}[2 + 2 \cos^2 2x] = \frac{1}{2}(1 + \cos^2 2x)$

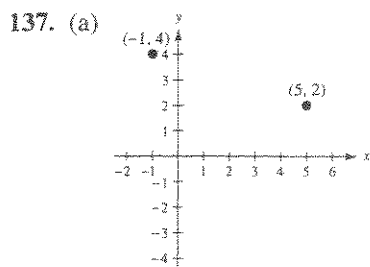
(c) $f(x) = \sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x - 2 \sin^2 x \cos^2 x$
 $= (\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x = 1 - 2 \sin^2 x \cos^2 x$

(d) $f(x) = 1 - \frac{1}{2} \sin^2 2x$

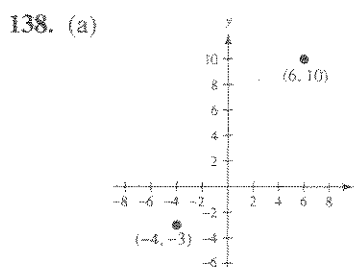
(e) Answers will vary.

135. Answers will vary.

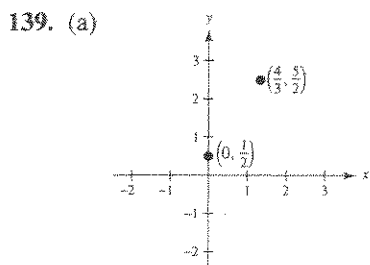
136. (a) Sample answer: $\cos(3\theta) = \cos(2\theta + \theta) = \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$
 $= (\cos^2 \theta - \sin^2 \theta) \cos \theta - 2 \sin \theta \cos \theta \sin \theta = \cos^3 \theta - 3 \sin^2 \theta \cos \theta$
- (b) Sample answer: $\cos 4\theta = 2 \cos^2 2\theta - 1 = 2(2 \cos^2 \theta - 1)^2 - 1 = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$



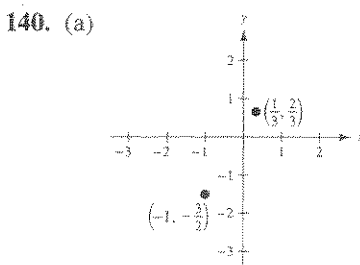
- (b) Distance:
 $\sqrt{(5 + 1)^2 + (2 - 4)^2} = \sqrt{40} = 2\sqrt{10}$
- (c) Midpoint: $\left(\frac{-1 + 5}{2}, \frac{4 + 2}{2}\right) = (2, 3)$



- (b) Distance: $\sqrt{(6 + 4)^2 + (10 + 3)^2} = \sqrt{269}$
- (c) Midpoint: $\left(\frac{6 - 4}{2}, \frac{10 - 3}{2}\right) = \left(1, \frac{7}{2}\right)$



- (b) Distance:
 $\sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{5}{2} - \frac{1}{2}\right)^2} = \sqrt{\frac{16}{9} + 4}$
 $= \frac{\sqrt{52}}{3} = \frac{2\sqrt{13}}{3}$
- (c) Midpoint: $\left(\frac{0 + (4/3)}{2}, \frac{(1/2) + (5/2)}{2}\right) = \left(\frac{2}{3}, \frac{3}{2}\right)$



- (b) Distance: $\sqrt{\left(\frac{1}{3} + 1\right)^2 + \left(\frac{2}{3} + \frac{3}{2}\right)^2} = \frac{\sqrt{233}}{6}$
- (c) Midpoint: $\left(\frac{(1/3) - 1}{2}, \frac{(2/3) - (3/2)}{2}\right) = \left(-\frac{1}{3}, -\frac{5}{12}\right)$

141. (a) Complement: $90^\circ - 55^\circ = 35^\circ$
 Supplement: $180^\circ - 55^\circ = 125^\circ$
- (b) Complement: None
 Supplement: $180^\circ - 162^\circ = 18^\circ$

142. (a) Complement: None
 Supplement: $180^\circ - 109^\circ = 71^\circ$
- (b) Complement: $90^\circ - 78^\circ = 12^\circ$
 Supplement: $180^\circ - 78^\circ = 102^\circ$

143. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{18} = \frac{8\pi}{18} = \frac{4\pi}{9}$
 Supplement: $\pi - \frac{\pi}{18} = \frac{17\pi}{18}$
- (b) Complement: $\frac{\pi}{2} - \frac{9\pi}{20} = \frac{\pi}{20}$
 Supplement: $\pi - \frac{9\pi}{20} = \frac{11\pi}{20}$

144. (a) Complement: $\frac{\pi}{2} - 0.95 \approx 0.6208$
 Supplement: $\pi - 0.95 \approx 2.1916$
- (b) Complement: None
 Supplement: $\pi - 2.76 \approx 0.3816$

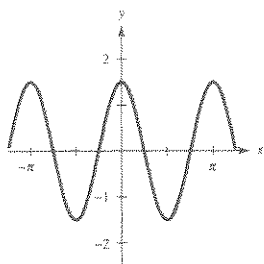
$$145. s = r\theta \Rightarrow \theta = \frac{s}{r} = \frac{7}{15} \approx 0.467 \text{ rad}$$

$$146. s = r\theta = 21(35^\circ)\left(\frac{\pi}{180^\circ}\right) \approx 12.8282 \text{ cm}$$

$$147. f(x) = \frac{3}{2} \cos(2x)$$

$$\text{Period: } \frac{2\pi}{2} = \pi$$

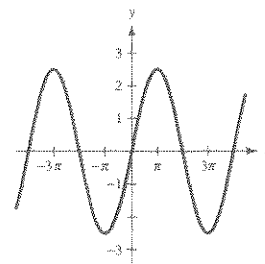
$$\text{Amplitude: } \frac{3}{2}$$



$$148. f(x) = \frac{5}{2} \sin \frac{x}{2}$$

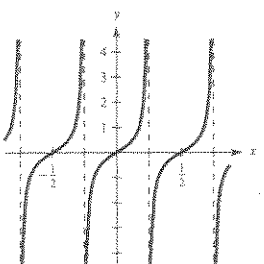
$$\text{Period: } 4\pi$$

$$\text{Amplitude: } \frac{5}{2}$$



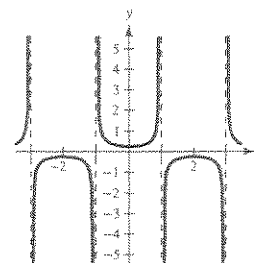
$$149. f(x) = \frac{1}{2} \tan(2\pi x)$$

$$\text{Period: } \frac{\pi}{2\pi} = \frac{1}{2}$$



$$150. f(x) = \frac{1}{4} \sec \frac{\pi x}{2}$$

$$\text{Period: } \frac{2\pi}{\pi/2} = 4$$



Review Exercises for Chapter 5

$$1. \frac{1}{\cos x} = \sec x$$

$$2. \frac{1}{\sin x} = \csc x$$

$$3. \frac{1}{\sec x} = \cos x$$

$$4. \frac{1}{\tan x} = \cot x$$

$$5. \sqrt{1 - \cos^2 x} = |\sin x|$$

$$6. \sqrt{1 + \tan^2 x} = |\sec x|$$

$$7. \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$8. \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$9. \sec(-x) = \sec x$$

$$10. \tan(-x) = -\tan x$$

$$11. \sin x = \frac{4}{5}, \cos x = \frac{3}{5}, \text{ Quadrant I}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{4}{3}$$

$$\cot x = \frac{3}{4}$$

$$\sec x = \frac{5}{3}$$

$$\csc x = \frac{5}{4}$$

$$12. \tan \theta = \frac{2}{3}, \sec \theta = \frac{\sqrt{13}}{3}, \text{ Quadrant I}$$

$$\cos \theta = \frac{3\sqrt{13}}{13}$$

$$\sin \theta = \tan \theta \cos \theta = \frac{2}{3} \left(\frac{3\sqrt{13}}{13} \right) = \frac{2\sqrt{13}}{13}$$

$$\csc \theta = \frac{\sqrt{13}}{2}$$

$$\cot \theta = \frac{3}{2}$$

$$13. \sin\left(\frac{\pi}{2} - x\right) = \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2},$$

$$\sin x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}, \quad \text{Quadrant IV}$$

$$\tan x = -1$$

$$\cot x = -1$$

$$\sec x = \sqrt{2}$$

$$\csc x = -\sqrt{2}$$

$$14. \csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta = 3, \sin \theta = \frac{2\sqrt{2}}{3}, \text{ Quadrant I}$$

$$\cos \theta = \frac{1}{3}$$

$$\tan \theta = 2\sqrt{2}$$

$$\cot \theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

$$\csc \theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$15. \frac{1}{\tan^2 x + 1} = \frac{1}{\sec^2 x} = \cos^2 x$$

$$16. \frac{\sec^2 x - 1}{\sec x - 1} = \frac{(\sec x - 1)(\sec x + 1)}{\sec x - 1} = \sec x + 1$$

$$17. \frac{\sin^2 \alpha - \cos^2 \alpha}{\sin^2 \alpha - \sin \alpha \cos \alpha} = \frac{(\sin \alpha + \cos \alpha)(\sin \alpha - \cos \alpha)}{\sin \alpha(\sin \alpha - \cos \alpha)} = \frac{\sin \alpha + \cos \alpha}{\sin \alpha} = 1 + \cot \alpha$$

$$18. \frac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} = \frac{(\sin \beta + \cos \beta)(\sin^2 \beta - \sin \beta \cos \beta + \cos^2 \beta)}{\sin \beta + \cos \beta} = 1 - \sin \beta \cos \beta$$

$$19. \tan^2 \theta (\csc^2 \theta - 1) = \tan^2 \theta (\cot^2 \theta)$$

$$= \tan^2 \theta \left(\frac{1}{\tan^2 \theta} \right) = 1$$

$$20. \csc^2 x (1 - \cos^2 x) = \csc^2 x (\sin^2 x) = 1$$

$$21. \tan\left(\frac{\pi}{2} - x\right) \sec x = \cot x \sec x$$

$$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} = \frac{1}{\sin x} = \csc x$$

$$22. \frac{\sin(-x) \cot x}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{(-\sin x) \left(\frac{\cos x}{\sin x} \right)}{\cos x} = -1$$

$$\begin{aligned} 23. \sin^{-1/2} x \cos x &= \frac{\cos x}{\sin^{1/2} x} \\ &= \frac{\cos x}{\sqrt{\sin x}} \cdot \frac{\sqrt{\sin x}}{\sqrt{\sin x}} \\ &= \frac{\cos x}{\sin x} \sqrt{\sin x} = \cot x \sqrt{\sin x} \end{aligned}$$

$$\begin{aligned} 24. \csc^2 x - \csc x \cot x &= \frac{1}{\sin^2 x} - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} \\ &= \frac{1 - \cos x}{\sin^2 x} \end{aligned}$$

$$\begin{aligned} 25. \cos x (\tan^2 x + 1) &= \cos x \sec^2 x \\ &= \frac{1}{\sec x} \sec^2 x = \sec x \end{aligned}$$

$$\begin{aligned} 26. \sec^2 x \cot x - \cot x &= \cot x (\sec^2 x - 1) \\ &= \cot x \tan^2 x \\ &= \frac{1}{\tan x} \tan^2 x = \tan x \end{aligned}$$

$$\begin{aligned} 27. \sin^3 \theta + \sin \theta \cos^2 \theta &= \sin \theta (\sin^2 \theta + \cos^2 \theta) \\ &= \sin \theta \end{aligned}$$

$$\begin{aligned} 28. \cot^2 x - \cos^2 x &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \\ &= \cos^2 x [\csc^2 x - 1] \\ &= \cos^2 x \cdot \cot^2 x \end{aligned}$$

29. $\sin^5 x \cos^2 x = \sin^4 x \cos^2 x \sin x$

$$= (1 - \cos^2 x)^2 \cos^2 x \sin x$$

$$= (1 - 2\cos^2 x + \cos^4 x) \cos^2 x \sin x$$

$$= (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x$$

30. $\cos^3 x \sin^2 x = \cos x (\cos^2 x) \sin^2 x$

$$= \cos x (1 - \sin^2 x) \sin^2 x$$

$$= (\sin^2 x - \sin^4 x) \cos x$$

31.
$$\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta}} = \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} = \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} = \frac{|1 - \sin \theta|}{|\cos \theta|} = \frac{1 - \sin \theta}{|\cos \theta|}$$

Note: We can drop the absolute value on $1 - \sin \theta$ since it is always nonnegative.

32.
$$\begin{aligned} \sqrt{1 - \cos x} &= \sqrt{(1 - \cos x) \frac{1 + \cos x}{1 + \cos x}} \\ &= \sqrt{\frac{\sin^2 x}{1 + \cos x}} = \frac{|\sin x|}{\sqrt{1 + \cos x}} \end{aligned}$$

33.
$$\frac{\csc(-x)}{\sec(-x)} = -\frac{\csc x}{\sec x} = -\frac{\cos x}{\sin x} = -\cot x$$

34.
$$\begin{aligned} \frac{1 + \sec(-x)}{\sin(-x) + \tan(-x)} &= \frac{1 + \sec x}{-\sin x - \tan x} \\ &= \frac{1 + \sec x}{-\sin x(1 + \sec x)} \\ &= -\frac{1}{\sin x} = -\csc x \end{aligned}$$

35.
$$\csc^2\left(\frac{\pi}{2} - x\right) - 1 = \sec^2 x - 1 = \tan^2 x$$

36.
$$\begin{aligned} \tan\left(\frac{\pi}{2} - x\right) \sec x &= \cot x \sec x \\ &= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \\ &= \frac{1}{\sin x} = \csc x \end{aligned}$$

37. $2 \sin x - 1 = 0$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + 2n\pi$$

$$x = \frac{5\pi}{6} + 2n\pi$$

38. $\tan x + 1 = 0$

$$\tan x = -1$$

$$x = \frac{3\pi}{4} + n\pi$$

39. $\sin x = \sqrt{3} - \sin x$

$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + 2n\pi$$

$$x = \frac{2\pi}{3} + 2n\pi$$

40. $4 \cos x = 1 + 2 \cos x$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2n\pi$$

$$x = \frac{5\pi}{3} + 2n\pi$$

41. $3\sqrt{3} \tan x = 3$

$$\tan x = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6} + n\pi$$

$$42. \frac{1}{2} \sec x - 1 = 0$$

$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2n\pi$$

$$x = \frac{5\pi}{3} + 2n\pi$$

$$43. 3 \csc^2 x = 4$$

$$\csc^2 x = \frac{4}{3}$$

$$\sin^2 x = \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + n\pi$$

$$x = \frac{2\pi}{3} + n\pi$$

$$44. 4 \tan^2 x - 1 = \tan^2 x$$

$$3 \tan^2 x = 1$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6} + n\pi$$

$$x = \frac{5\pi}{6} + n\pi$$

$$45. 4 \cos^2 x - 3 = 0$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6} + n\pi$$

$$x = \frac{5\pi}{6} + n\pi$$

$$46. \sin x(\sin x + 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \sin x = -1$$

$$x = n\pi \quad \text{or} \quad x = \frac{3\pi}{2} + 2n\pi$$

$$47. \sin x - \tan x = 0$$

$$\sin x - \frac{\sin x}{\cos x} = 0$$

$$\sin x \cos x - \sin x = 0$$

$$\sin x(\cos x - 1) = 0$$

$$\sin x = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$x = n\pi \quad \cos x = 1$$

$$48. \csc x - 2 \cot x = 0$$

$$\frac{1}{\sin x}(1 - 2 \cos x) = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} + 2n\pi$$

$$x = \frac{5\pi}{3} + 2n\pi$$

$$49. 2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$2 \cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad x = 0$$

$$50. 2 \sin^2 x - 3 \sin x + 1 = 0$$

$$(2 \sin x - 1)(\sin x - 1) = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = 1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad \text{or} \quad x = \frac{\pi}{2}$$

51. $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x = 1$

$\sin x(\sin x - 1) = 0$

$\sin x = 0 \quad \text{or} \quad \sin x = 1$

$x = 0, \pi \quad x = \frac{\pi}{2}$

53. $2 \sin 2x = \sqrt{2}$

$\sin 2x = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

55. $\cos 4x(\cos x - 1) = 0$

$\cos 4x = 0 \quad \text{or} \quad \cos x - 1 = 0$

$4x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \frac{13\pi}{2}, \frac{15\pi}{2}$

$\text{or} \quad \cos x = 1$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}, 0$

57. $\cos 4x - 7 \cos 2x = 8$

$2 \cos^2 2x - 1 - 7 \cos 2x = 8$

$2 \cos^2 2x - 7 \cos 2x - 9 = 0$

$(2 \cos 2x - 9)(\cos 2x + 1) = 0$

$2 \cos 2x - 9 = 0 \quad \text{or} \quad \cos 2x + 1 = 0$

$\cos 2x = \frac{9}{2} \quad \cos 2x = -1$

No solution

$2x = \pi + 2n\pi$

$x = \frac{\pi}{2} + n\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

52. $\sin^2 x + 2 \cos x = 2$

$(1 - \cos^2 x) + 2 \cos x - 2 = 0$

$\cos^2 x - 2 \cos x + 1 = 0$

$(\cos x - 1)^2 = 0$

$\cos x = 1$

$x = 0$

54. $\sqrt{3} \tan 3x = 0$

$\tan 3x = 0$

$3x = k\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

56. $3 \csc^2 5x = -4$

$\csc^2 5x = -\frac{4}{3}$

No solutions

58. $\sin 4x - \sin 2x = 0$

$2 \cos 3x \sin x = 0$

$\cos 3x = 0 \quad \text{or} \quad \sin x = 0$

$3x = \frac{\pi}{2} + n\pi \quad x = 0, \pi$

$x = \frac{\pi}{6} + \frac{n\pi}{3}$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

59. $2 \sin 2x - 1 = 0$

$$\sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + 2n\pi$$

$$x = \frac{\pi}{12} + n\pi \quad \text{or} \quad x = \frac{5\pi}{12} + n\pi$$

60. $2 \cos 4x + \sqrt{3} = 0$

$$\cos 4x = \frac{-\sqrt{3}}{2}$$

$$4x = \frac{5\pi}{6} + 2n\pi \quad \text{or} \quad 4x = \frac{7\pi}{6} + 2n\pi$$

$$x = \frac{5\pi}{24} + \frac{n\pi}{2} \quad \text{or} \quad x = \frac{7\pi}{24} + \frac{n\pi}{2}$$

61. $2 \sin^2 3x - 1 = 0$

$$\sin^2 3x = \frac{1}{2}$$

$$\sin 3x = \pm \frac{\sqrt{2}}{2}$$

$$3x = \frac{\pi}{4} + \frac{n\pi}{2}$$

$$x = \frac{\pi}{12} + \frac{n\pi}{6}$$

62. $4 \cos^2 2x - 3 = 0$

$$\cos^2 2x = \frac{3}{4}$$

$$\cos 2x = \pm \frac{\sqrt{3}}{2}$$

$$2x = \frac{\pi}{6} + n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2} \quad \text{or} \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

63. $\sin^2 x - 2 \sin x = 0$

$$\sin x(\sin x - 2) = 0$$

$$\sin x = 0 \quad \text{or} \quad \sin x = 2 \quad (\text{impossible})$$

$$x = 0, \pi$$

64. $3 \cos^2 x + 5 \cos x = 0$

$$\cos x(3 \cos x + 5) = 0$$

$$\cos x = 0 \implies x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = -\frac{5}{3} \quad (\text{impossible})$$

65. $\tan^2 \theta + 3 \tan \theta - 10 = 0$

$$(\tan \theta + 5)(\tan \theta - 2) = 0$$

$$\tan \theta = -5 \implies$$

$$\theta = \arctan(-5) + \pi, \arctan(-5) + 2\pi$$

$$\tan \theta = 2 \implies \theta = \arctan(2), \arctan(2) + \pi$$

$$\theta \approx 1.1071, 1.7682, 4.2487, 4.9098$$

66. $\sec^2 x + 6 \tan x + 4 = 0$

$$(1 + \tan^2 x) + 6 \tan x + 4 = 0$$

$$\tan^2 x + 6 \tan x + 5 = 0$$

$$(\tan x + 1)(\tan x + 5) = 0$$

$$\tan x = -1 \quad \text{or} \quad \tan x = -5$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4} \quad \text{or} \quad x \approx 1.7682, 4.9098$$

67. $\sin 285^\circ = \sin(315^\circ - 30^\circ)$

$$= \sin 315^\circ \cos 30^\circ - \cos 315^\circ \sin 30^\circ$$

$$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos 285^\circ = \cos(315^\circ - 30^\circ) = \cos 315^\circ \cos 30^\circ + \sin 315^\circ \sin 30^\circ$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\tan 285^\circ = -\frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = -2 - \sqrt{3}$$

$$68. \sin 345^\circ = \sin(300^\circ + 45^\circ) = \sin 300^\circ \cos 45^\circ + \cos 300^\circ \sin 45^\circ = -\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cos 345^\circ = \cos(300^\circ + 45^\circ) = \cos 300^\circ \cos 45^\circ - \sin 300^\circ \sin 45^\circ = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\tan 345^\circ = \frac{\sin 345^\circ}{\cos 345^\circ} = \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} + \sqrt{6}} = \sqrt{3} - 2$$

$$69. \sin \frac{31\pi}{12} = \sin\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \sin \frac{11\pi}{6} \cos \frac{3\pi}{4} + \sin \frac{3\pi}{4} \cos \frac{11\pi}{6}$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\cos \frac{31\pi}{12} = \cos\left(\frac{11\pi}{6} + \frac{3\pi}{4}\right) = \cos \frac{11\pi}{6} \cos \frac{3\pi}{4} - \sin \frac{11\pi}{6} \sin \frac{3\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\tan \frac{31\pi}{12} = \frac{\sin(31\pi/12)}{\cos(31\pi/12)} = \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}} = -2 - \sqrt{3}$$

$$70. \sin\left(\frac{13\pi}{12}\right) = \sin\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right) = \sin \frac{11\pi}{6} \cos \frac{3\pi}{4} - \sin \frac{3\pi}{4} \cos \frac{11\pi}{6}$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$\cos\left(\frac{13\pi}{12}\right) = \cos\left(\frac{11\pi}{6} - \frac{3\pi}{4}\right) = \cos \frac{11\pi}{6} \cos \frac{3\pi}{4} + \sin \frac{11\pi}{6} \sin \frac{3\pi}{4}$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$$

$$\tan\left(\frac{13\pi}{12}\right) = \frac{\sin(13\pi/12)}{\cos(13\pi/12)} = \frac{\sqrt{2} - \sqrt{6}}{-\sqrt{6} - \sqrt{2}} = 2 - \sqrt{3}$$

$$71. \sin 130^\circ \cos 50^\circ + \cos 130^\circ \sin 50^\circ = \sin(130^\circ + 50^\circ) = \sin 180^\circ = 0$$

$$72. \cos 45^\circ \cos 120^\circ - \sin 45^\circ \sin 120^\circ = \cos(45^\circ + 120^\circ) = \cos(165^\circ)$$

$$73. \frac{\tan 25^\circ + \tan 10^\circ}{1 - \tan 25^\circ \tan 10^\circ} = \tan(25^\circ + 10^\circ) = \tan 35^\circ$$

$$74. \frac{\tan 63^\circ - \tan 118^\circ}{1 + \tan 63^\circ \tan 118^\circ} = \tan(63^\circ - 118^\circ) = \tan(-55^\circ) = -\tan(55^\circ)$$

$$\text{For Exercises 75-80, } \sin u = \frac{3}{5}, \cos v = -\frac{7}{25}, \cos u = -\frac{4}{5}, \sin v = \frac{24}{25}.$$

$$75. \sin(u + v) = \sin u \cos v + \sin v \cos u$$

$$= \frac{3}{5}\left(-\frac{7}{25}\right) + \frac{24}{25}\left(-\frac{4}{5}\right) = -\frac{117}{125}$$

$$76. \tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$= \frac{(-3/4) + (-24/7)}{1 - (-3/4)(-24/7)} = \frac{117}{44}$$

$$77. \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$= \frac{(-3/4) - (-24/7)}{1 + (-3/4)(-24/7)} = \frac{3}{4}$$

$$79. \cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) - \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = -\frac{44}{125}$$

$$81. \sin^{-1} 0 = 0 \text{ and } \cos^{-1}(-1) = \pi$$

$$\sin(\sin^{-1} 0 + \cos^{-1}(-1)) = \sin(0 + \pi) = 0$$

$$83. \cos^{-1} 1 = 0 \text{ and } \sin^{-1}(-1) = -\frac{\pi}{2}$$

$$\cos(\cos^{-1} 1 - \sin^{-1}(-1)) = \cos\left(0 + \frac{\pi}{2}\right) = 0$$

$$85. \cos\left(x + \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} - \sin x \sin \frac{\pi}{2}$$

$$= (\cos x)(0) - (\sin x)(1) = -\sin x$$

$$87. \cot\left(\frac{\pi}{2} - x\right) = \frac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]}$$

$$= \frac{\cos(\pi/2) \cos x + \sin(\pi/2) \sin x}{\sin(\pi/2) \cos x - \sin x \cos(\pi/2)}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

$$89. \cos 3x = \cos(2x + x)$$

$$= \cos 2x \cos x - \sin 2x \sin x$$

$$= (\cos^2 x - \sin^2 x) \cos x - 2 \sin x \cos x \sin x$$

$$= \cos^3 x - 3 \sin^2 x \cos x$$

$$= \cos^3 x - 3 \cos x(1 - \cos^2 x)$$

$$= \cos^3 x - 3 \cos x + 3 \cos^3 x$$

$$= 4 \cos^3 x - 3 \cos x$$

$$91. \sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{2}$$

$$2 \cos x \sin \frac{\pi}{2} = \sqrt{2} \quad (\text{Sum-to-Product})$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$78. \sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$= \frac{3}{5}\left(-\frac{7}{25}\right) - \left(-\frac{4}{5}\right)\left(\frac{24}{25}\right) = \frac{3}{5}$$

$$80. \cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{24}{25}\right) = \frac{4}{5}$$

$$82. \cos^{-1} 1 = 0 \text{ and } \sin^{-1} 0 = 0$$

$$\cos(\cos^{-1} 1 + \sin^{-1} 0) = \cos(0 + 0) = 1$$

$$84. \cos^{-1}(-1) = \pi \text{ and } \cos^{-1} 1 = 0$$

$$\tan(\cos^{-1}(-1) + \cos^{-1} 1) = \tan(\pi + 0) = 0$$

$$86. \sin\left(x - \frac{3\pi}{2}\right) = \sin x \cos \frac{3\pi}{2} - \sin \frac{3\pi}{2} \cos x$$

$$= (\sin x)(0) - (-1)(\cos x) = \cos x$$

$$88. \sin(\pi - x) = \sin \pi \cos x - \sin x \cos \pi$$

$$= (0)(\cos x) - (\sin x)(-1)$$

$$= \sin x$$

$$90. \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$= \frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}$$

$$= \tan \alpha + \tan \beta$$

$$92. \cos\left(x + \frac{\pi}{4}\right) - \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$-2 \sin x \sin \frac{\pi}{4} = 1 \quad (\text{Sum-to-Product})$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

93. $\sin u = -\frac{5}{7}$, $\pi < u < \frac{3\pi}{2}$, Quadrant III

$$\cos^2 u = 1 - \left(-\frac{5}{7}\right)^2 = \frac{24}{49} \Rightarrow \cos u = -\frac{2\sqrt{6}}{7}$$

$$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{5}{7}\right)\left(-\frac{2\sqrt{6}}{7}\right) = \frac{20\sqrt{6}}{49}$$

$$\cos 2u = 1 - 2 \sin^2 u$$

$$= 1 - 2\left(-\frac{5}{7}\right)^2 = 1 - \frac{50}{49} = -\frac{1}{49}$$

$$\tan 2u = \frac{\sin 2u}{\cos 2u} = \frac{20\sqrt{6}}{-1} = -20\sqrt{6}$$

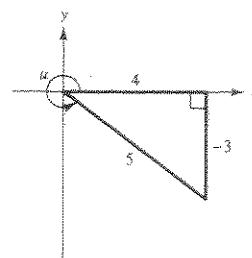
94. $\cos u = \frac{4}{5}$, $\frac{3\pi}{2} < u < 2\pi$, Quadrant IV

$$\sin u = -\frac{3}{5}, \tan u = -\frac{3}{4}$$

$$\sin 2u = 2 \sin u \cos u = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2u = \frac{\sin 2u}{\cos 2u} = \frac{-24}{7}$$



95. $\tan u = -\frac{2}{9}$, $\frac{\pi}{2} < u < \pi$, Quadrant II

$$\sec^2 u = \tan^2 u + 1 = \frac{4}{81} + 1 = \frac{85}{81} \Rightarrow$$

$$\sec u = -\frac{\sqrt{85}}{9}$$

$$\cos u = -\frac{9\sqrt{85}}{85}, \sin u = (\tan u)(\cos u) = \frac{2\sqrt{85}}{85}$$

$$\sin 2u = 2 \sin u \cos u$$

$$= 2\left(\frac{2\sqrt{85}}{85}\right)\left(-\frac{9\sqrt{85}}{85}\right) = -\frac{36}{85}$$

$$\cos 2u = 1 - 2 \sin^2 u = 1 - 2\left(\frac{4}{85}\right) = \frac{77}{85}$$

$$\tan 2u = \frac{\sin 2u}{\cos 2u} = -\frac{36}{77}$$

96. $\cos u = -\frac{2}{\sqrt{5}}$, $\frac{\pi}{2} < u < \pi$, Quadrant II

$$\sin^2 u = 1 - \cos^2 u = 1 - \frac{4}{5} = \frac{1}{5} \Rightarrow \sin u = \frac{1}{\sqrt{5}}$$

$$\sin 2u = 2 \sin u \cos u = 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = -\frac{4}{5}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

$$\tan 2u = \frac{\sin 2u}{\cos 2u} = -\frac{4}{3}$$

97. $6 \sin x \cos x = 3[2 \sin x \cos x] = 3 \sin 2x$

99. $1 - 4 \sin^2 x \cos^2 x = 1 - (2 \sin x \cos x)^2$
 $= 1 - \sin^2 2x = \cos^2 2x$

98. $4 \sin x \cos x + 2 = 2(2 \sin x \cos x) + 2$
 $= 2 \sin 2x + 2$

100. $\sin 4x = 2 \sin 2x \cos 2x$
 $= 2[2 \sin x \cos x(\cos^2 x - \sin^2 x)]$
 $= 4 \sin x \cos x(2 \cos^2 x - 1)$
 $= 8 \cos^3 x \sin x - 4 \cos x \sin x$

$$\begin{aligned}
 101. \quad r &= \frac{1}{32}v_0^2 \sin 2\theta \\
 100 &= \frac{1}{32}(80)^2 \sin 2\theta \\
 \sin 2\theta &= 0.5
 \end{aligned}$$

$$2\theta = 30^\circ \quad \text{or} \quad 2\theta = 180^\circ - 30^\circ = 150^\circ$$

$$\theta = 15^\circ \quad \theta = 75^\circ$$

$$\begin{aligned}
 102. \quad r &= \frac{1}{32}v_0^2 \sin 2\theta \\
 77 &= \frac{1}{32}(50)^2 \sin 2\theta \\
 \sin 2\theta &= 0.9856 \\
 2\theta &= 80.2649^\circ \quad \text{or} \quad 99.7351^\circ \\
 \theta &= 40.13^\circ \quad \text{or} \quad 49.87^\circ
 \end{aligned}$$

$$\begin{aligned}
 103. \quad \sin^6 x &= \left(\frac{1 - \cos 2x}{2} \right)^3 = \frac{1}{8}(1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x) \\
 &= \frac{1}{8} \left[1 - 3 \cos 2x + 3 \left(\frac{1 + \cos 4x}{2} \right) - \cos 2x \left(\frac{1 + \cos 4x}{2} \right) \right] \\
 &= \frac{1}{8} \left(1 - 3 \cos 2x + \frac{3}{2} + \frac{3}{2} \cos 4x - \frac{1}{2} \cos 2x - \frac{1}{2} \cos 2x \cos 4x \right) \\
 &= \frac{1}{16} \left(5 - 7 \cos 2x + 3 \cos 4x - \frac{1}{2} [\cos 2x + \cos 6x] \right) \\
 &= \frac{1}{32} (10 - 15 \cos 2x + 6 \cos 4x - \cos 6x)
 \end{aligned}$$

$$\begin{aligned}
 104. \quad \cos^4 x \sin^4 x &= \left(\frac{1 + \cos 2x}{2} \right)^2 \left(\frac{1 - \cos 2x}{2} \right)^2 \\
 &= \frac{(1 + 2 \cos 2x + \cos^2 2x)(1 - 2 \cos 2x + \cos^2 2x)}{16} \\
 &= \frac{\left(1 + 2 \cos 2x + \frac{1 + \cos 4x}{2} \right) \left(1 - 2 \cos 2x + \frac{1 + \cos 4x}{2} \right)}{16} \\
 &= \frac{(3 + 4 \cos 2x + \cos 4x)(3 - 4 \cos 2x + \cos 4x)}{64} \\
 &= \frac{1}{64} [(3 + \cos 4x) + 4 \cos 2x][(3 + \cos 4x) - 4 \cos 2x] \\
 &= \frac{1}{64} [(3 + \cos 4x)^2 - 16 \cos^2 2x] \\
 &= \frac{1}{64} [9 + 6 \cos 4x + \cos^2 4x - 16 \cos^2 2x] \\
 &= \frac{1}{64} \left[9 + 6 \cos 4x + \frac{1 + \cos 8x}{2} - 16 \cdot \frac{1 + \cos 4x}{2} \right] \\
 &= \frac{1}{64} \left[\frac{3}{2} + \frac{1}{2} \cos 8x - 2 \cos 4x \right] \\
 &= \frac{1}{128} (\cos 8x - 4 \cos 4x + 3)
 \end{aligned}$$

$$105. \cos^4 2x = \left(\frac{1 + \cos 4x}{2} \right)^2$$

$$= \frac{1}{4}(1 + 2\cos 4x + \cos^2 4x)$$

$$= \frac{1}{4}\left(1 + 2\cos 4x + \frac{1 + \cos 8x}{2}\right)$$

$$= \frac{1}{8}(2 + 4\cos 4x + 1 + \cos 8x)$$

$$= \frac{1}{8}(3 + 4\cos 4x + \cos 8x)$$

$$106. \sin^4 2x = \left(\frac{1 - \cos 4x}{2} \right)^2$$

$$= \frac{1}{4}[1 - 2\cos 4x + \cos^2 4x]$$

$$= \frac{1}{4}\left[1 - 2\cos 4x + \frac{1 + \cos 8x}{2}\right]$$

$$= \frac{1}{4}\left[\frac{3}{2} - 2\cos 4x + \frac{1}{2}\cos 8x\right]$$

$$= \frac{1}{8}(3 + \cos 8x - 4\cos 4x)$$

$$107. \sin 105^\circ = \sin\left(\frac{1}{2} \cdot 210^\circ\right) = \sqrt{\frac{1 - \cos 210^\circ}{2}} = \sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\cos 105^\circ = \cos\left(\frac{1}{2} \cdot 210^\circ\right) = -\sqrt{\frac{1 + \cos 210^\circ}{2}} = -\sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{-\sqrt{2 - \sqrt{3}}}{2}$$

$$\tan 105^\circ = \tan\left(\frac{1}{2} \cdot 210^\circ\right) = \frac{\sin 210^\circ}{1 + \cos 210^\circ} = \frac{-1/2}{1 - (\sqrt{3}/2)} = \frac{1}{\sqrt{3} - 2} = -2 - \sqrt{3}$$

$$108. 112^\circ 30' = 112.5^\circ$$

$$\sin(112.5^\circ) = \sin\left(\frac{1}{2} \cdot 225^\circ\right) = \sqrt{\frac{1 - \cos 225^\circ}{2}} = \sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\cos(112.5^\circ) = \cos\left(\frac{1}{2} \cdot 225^\circ\right) = -\sqrt{\frac{1 + \cos 225^\circ}{2}} = -\sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = \frac{-\sqrt{2 - \sqrt{2}}}{2}$$

$$\begin{aligned} \tan(112.5^\circ) &= \tan\left(\frac{1}{2} \cdot 225^\circ\right) = \frac{\sin 225^\circ}{1 + \cos 225^\circ} = \frac{-\sqrt{2}/2}{1 - (\sqrt{2}/2)} = \frac{-\sqrt{2}}{2 - \sqrt{2}} \cdot \frac{2 + \sqrt{2}}{2 + \sqrt{2}} \\ &= \frac{-2 - 2\sqrt{2}}{2} = -1 - \sqrt{2} \end{aligned}$$

$$109. \sin\left(\frac{7\pi}{8}\right) = \sin\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = \sqrt{\frac{1 - \cos(7\pi/4)}{2}} = \sqrt{\frac{1 - (\sqrt{2}/2)}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$\cos\left(\frac{7\pi}{8}\right) = \cos\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = -\sqrt{\frac{1 + \cos(7\pi/4)}{2}} = -\sqrt{\frac{1 + (\sqrt{2}/2)}{2}} = \frac{-\sqrt{2 + \sqrt{2}}}{2}$$

$$\tan\left(\frac{7\pi}{8}\right) = \tan\left(\frac{1}{2} \cdot \frac{7\pi}{4}\right) = \frac{\sin(7\pi/4)}{1 + \cos(7\pi/4)} = \frac{-\sqrt{2}/2}{1 + (\sqrt{2}/2)} = \frac{-\sqrt{2}}{2 + \sqrt{2}} = 1 - \sqrt{2}$$

$$110. \frac{11\pi}{12} = \frac{1}{2}\left(\frac{11\pi}{6}\right), \text{ Quadrant II}$$

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = \sqrt{\frac{1 - \cos(11\pi/6)}{2}} = \sqrt{\frac{1 - (\sqrt{3}/2)}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = -\sqrt{\frac{1 + \cos(11\pi/6)}{2}} = -\sqrt{\frac{1 + (\sqrt{3}/2)}{2}} = \frac{-\sqrt{2 + \sqrt{3}}}{2}$$

$$\tan\left(\frac{11\pi}{12}\right) = \tan\left(\frac{1}{2} \cdot \frac{11\pi}{6}\right) = \frac{\sin(11\pi/6)}{1 + \cos(11\pi/6)} = \frac{-1/2}{1 + (\sqrt{3}/2)} = \frac{-1}{2 + \sqrt{3}} = -2 + \sqrt{3}$$

$$111. \sin u = \frac{3}{5}, 0 < u < \frac{\pi}{2} \Rightarrow \cos u = \frac{4}{5}$$

$$\begin{aligned}\sin\left(\frac{u}{2}\right) &= \sqrt{\frac{1 - \cos u}{2}} \\ &= \sqrt{\frac{1 - (4/5)}{2}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{u}{2}\right) &= \sqrt{\frac{1 + \cos u}{2}} \\ &= \sqrt{\frac{1 + (4/5)}{2}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}\end{aligned}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 - (4/5)}{3/5} = \frac{1}{3}$$

$$112. \tan u = \frac{21}{20}, \pi < u < \frac{3\pi}{2}, \text{ Quadrant III}$$

$$\sec^2 u = \tan^2 u + 1 = \frac{441}{400} + 1 = \frac{841}{400} \Rightarrow$$

$$\sec u = \frac{-29}{20}$$

$$\cos u = \frac{-20}{29}$$

$$\sin u = \cos u \tan u = \frac{-20}{29} \cdot \frac{21}{20} = \frac{-21}{29}$$

$$\sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (20/29)}{2}} = \frac{7\sqrt{58}}{58}$$

$$\begin{aligned}\cos \frac{u}{2} &= -\sqrt{\frac{1 + \cos u}{2}} = -\sqrt{\frac{1 - (20/29)}{2}} \\ &= \frac{-3\sqrt{58}}{58}\end{aligned}$$

$$\tan \frac{u}{2} = \frac{\sin(u/2)}{\cos(u/2)} = -\frac{7}{3}$$

$$113. \cos u = -\frac{2}{7}, \frac{\pi}{2} < u < \pi \Rightarrow \sin u = \sqrt{1 - \frac{4}{49}} = \frac{\sqrt{45}}{7}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (2/7)}{2}} = \sqrt{\frac{9}{14}} = \frac{3\sqrt{14}}{14}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (2/7)}{2}} = \sqrt{\frac{5}{14}} = \frac{\sqrt{70}}{14}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + (2/7)}{\sqrt{45}/7} = \frac{9}{\sqrt{45}} = \frac{9\sqrt{45}}{45} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

$$114. \sec u = -6, \frac{\pi}{2} < u < \pi, \text{ Quadrant II}$$

$$\cos u = -\frac{1}{6}, \sin u = \frac{\sqrt{35}}{6}, \tan u = -\sqrt{35}$$

$$\sin\left(\frac{u}{2}\right) = \sqrt{\frac{1 - \cos u}{2}} = \sqrt{\frac{1 + (1/6)}{2}} = \sqrt{\frac{7}{12}} = \frac{\sqrt{21}}{6}$$

$$\cos\left(\frac{u}{2}\right) = \sqrt{\frac{1 + \cos u}{2}} = \sqrt{\frac{1 - (1/6)}{2}} = \sqrt{\frac{5}{12}} = \frac{\sqrt{15}}{6}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{1 + (1/6)}{\sqrt{35}/6} = \frac{7}{\sqrt{35}} = \frac{\sqrt{35}}{5}$$

Note: $u \approx 99.6^\circ$

$$115. -\sqrt{\frac{1 + \cos 8x}{2}} = -|\cos 4x|$$

$$116. \frac{\sin 10x}{1 + \cos 10x} = \tan 5x$$

117. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.

$$V = A \cdot l$$

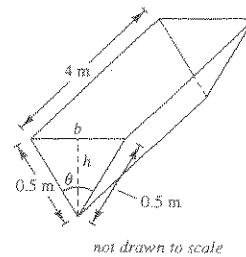
$$A = \frac{1}{2}bh$$

$$\cos \frac{\theta}{2} = \frac{h}{0.5} \Rightarrow h = 0.5 \cos \frac{\theta}{2}$$

$$\sin \frac{\theta}{2} = \frac{b/2}{0.5} \Rightarrow \frac{b}{2} = 0.5 \sin \frac{\theta}{2}$$

$$A = 0.5 \sin \frac{\theta}{2} \cdot 0.5 \cos \frac{\theta}{2} = (0.5)^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 0.25 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ square meters}$$

$$V = (0.25)(4) \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ cubic meters} = \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ cubic meters}$$



118. Volume V of the trough will be the area A of the isosceles triangle times the length l of the trough.

$$V = A \cdot l$$

$$A = \frac{1}{2}bh$$

$$\cos \frac{\theta}{2} = \frac{h}{0.5} \Rightarrow h = 0.5 \cos \frac{\theta}{2}$$

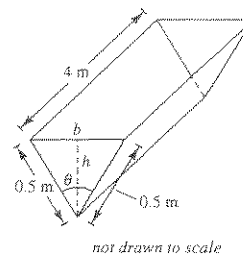
$$\sin \frac{\theta}{2} = \frac{b/2}{0.5} \Rightarrow \frac{b}{2} = 0.5 \sin \frac{\theta}{2}$$

$$A = 0.5 \sin \frac{\theta}{2} \cdot 0.5 \cos \frac{\theta}{2} = (0.5)^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = 0.25 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ square meters}$$

$$V = (0.25)(4) \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ cubic meters} = \sin \frac{\theta}{2} \cos \frac{\theta}{2} \text{ cubic meters}$$

$$V = \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{1}{2} \left(2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right) = \frac{1}{2} \sin \theta \text{ cubic meters}$$

Volume is maximum when $\theta = \pi/2$.



$$119. 6 \sin \frac{\pi}{4} \cos \frac{\pi}{4} = 6 \left[\frac{1}{2} \sin \left(\frac{\pi}{4} + \frac{\pi}{4} \right) + \sin \left(\frac{\pi}{4} - \frac{\pi}{4} \right) \right] = 3 \left(\sin \frac{\pi}{2} + \sin 0 \right) = 3$$

$$120. 4 \sin 15^\circ \sin 45^\circ = 4 \cdot \frac{1}{2} [\cos(15^\circ - 45^\circ) - \cos(15^\circ + 45^\circ)] = 2 [\cos(-30^\circ) - \cos(60^\circ)] \\ = 2 [\cos(30^\circ) - \cos(60^\circ)]$$

$$121. \sin 5\alpha \sin 4\alpha = \frac{1}{2} [\cos(5\alpha - 4\alpha) - \cos(5\alpha + 4\alpha)] = \frac{1}{2} [\cos \alpha - \cos 9\alpha]$$

$$122. \cos 6\theta \sin 8\theta = \frac{1}{2} [\sin(6\theta + 8\theta) - \sin(6\theta - 8\theta)] = \frac{1}{2} [\sin 14\theta - \sin(-2\theta)] = \frac{1}{2} [\sin 14\theta + \sin 2\theta]$$

$$123. \cos 5\theta + \cos 4\theta = 2 \cos \left(\frac{9\theta}{2} \right) \cos \left(\frac{\theta}{2} \right)$$

$$124. \sin 3\theta + \sin 2\theta = 2 \sin \left(\frac{5\theta}{2} \right) \cos \left(\frac{\theta}{2} \right)$$

$$125. \sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 2 \cos x \sin \frac{\pi}{4} = \sqrt{2} \cos x$$

$$126. \cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = -2 \sin\left(\frac{x + \frac{\pi}{6} + x - \frac{\pi}{6}}{2}\right) \sin\left(\frac{x + \frac{\pi}{6} - x + \frac{\pi}{6}}{2}\right) = -2 \sin x \sin \frac{\pi}{6}$$

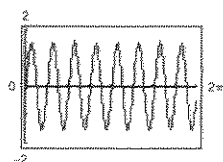
$$127. y = 1.5 \sin 8t - 0.5 \cos 8t$$

$$a = \frac{3}{2}, b = -\frac{1}{2}, B = 8, C = \arctan\left(-\frac{1/2}{3/2}\right)$$

$$y = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \sin\left(8t + \arctan\left(-\frac{1}{3}\right)\right)$$

$$y = \frac{1}{2}\sqrt{10} \sin\left(8t - \arctan \frac{1}{3}\right)$$

$$128. y = \frac{1}{2}\sqrt{10} \sin\left(8t - \arctan \frac{1}{3}\right)$$



$$129. \text{The amplitude is } \frac{\sqrt{10}}{2}.$$

$$130. \text{Frequency} = \frac{1}{\text{period}} = \frac{4}{\pi}$$

$$131. \text{If } \frac{\pi}{2} < \theta < \pi, \text{ then } \cos \frac{\theta}{2} < 0. \text{ False, if}$$

$$\frac{\pi}{2} < \theta < \pi \Rightarrow \frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2},$$

$$\text{which is in Quadrant I} \Rightarrow \cos\left(\frac{\theta}{2}\right) > 0.$$

$$132. \sin(x + y) = \sin x + \sin y. \text{ False.}$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$133. 4 \sin(-x) \cos(-x) = -2 \sin 2x. \text{ True.}$$

$$4 \sin(-x) \cos(-x) = 4(-\sin x)(\cos x) = -4 \sin x \cos x = -2(2 \sin x \cos x) = -2 \sin 2x$$

$$134. 4 \sin 45^\circ \cos 15^\circ = 1 + \sqrt{3}. \text{ True.}$$

$$\begin{aligned} 4 \sin 45^\circ \cos 15^\circ &= 4 \left(\frac{1}{2} [\sin(45^\circ + 15^\circ) + \sin(45^\circ - 15^\circ)] \right) = 2[\sin 60^\circ + \sin 30^\circ] \\ &= 2 \left[\frac{\sqrt{3}}{2} + \frac{1}{2} \right] = 2 \left(\frac{\sqrt{3} + 1}{2} \right) = 1 + \sqrt{3} \end{aligned}$$

$$135. \text{Answers will vary. See page 352.}$$

$$136. \text{No. } \cos \theta = \pm \sqrt{1 - \sin^2 \theta}$$

$$137. y_1 = \sec^2\left(\frac{\pi}{2} - x\right) = \csc^2 x$$

$$y_2 = \cot^2 x$$

$$\csc^2 x = \cot^2 x + 1$$

$$\text{Let } y_3 = y_2 + 1 = \cot^2 x + 1 = y_1.$$

$$138. y_1 = \frac{\cos 3x}{\cos x}$$

$$y_2 = (2 \sin x)^2$$

$$\text{From the graphs, } y_3 = -y_2 + 1 = y_1.$$

Chapter 5 Practice Test

- Find the value of the other five trigonometric functions, given $\tan x = \frac{4}{11}$, $\sec x < 0$.
- Simplify $\frac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)}$.
- Rewrite as a single logarithm and simplify $\ln|\tan \theta| - \ln|\cot \theta|$.
- True or false: $\cos\left(\frac{\pi}{2} - x\right) = \frac{1}{\csc x}$
- Factor and simplify: $\sin^4 x + (\sin^2 x) \cos^2 x$
- Multiply and simplify: $(\csc x + 1)(\csc x - 1)$
- Rationalize the denominator and simplify:
$$\frac{\cos^2 x}{1 - \sin x}$$
- Verify:
$$\tan^4 x + 2 \tan^2 x + 1 = \sec^4 x$$
- Verify: $\tan\left(\theta + \frac{\pi}{4}\right) = \frac{1 + \tan \theta}{1 - \tan \theta}$
- Use the sum or difference formulas to determine:
(a) $\sin 105^\circ$ (b) $\tan 15^\circ$
- Write $\sin(\arcsin x - \arccos x)$ as an algebraic expression in x .
- Use the double-angle formulas to determine:
(a) $\cos 120^\circ$ (b) $\tan 300^\circ$
- Use the half-angle formulas to determine:
(a) $\sin 22.5^\circ$ (b) $\tan \frac{\pi}{12}$
- Given $\sin \theta = 4/5$, θ lies in Quadrant II, find $\cos \theta/2$.
- Use the power-reducing identities to write $(\sin^2 x) \cos^2 x$ in terms of the first power of cosine.
- Rewrite as a sum: $6(\sin 5\theta) \cos 2\theta$
- Rewrite as a product: $\sin(x + \pi) + \sin(x - \pi)$
- Verify: $\frac{\sin 9x + \sin 5x}{\cos 9x - \cos 5x} = -\cot 2x$
- Find all solutions in the interval $[0, 2\pi)$:
 $4 \sin^2 x = 1$
- Find all solutions in the interval $[0, 2\pi)$:
 $\tan^2 \theta + (\sqrt{3} - 1) \tan \theta - \sqrt{3} = 0$
- Find all solutions in the interval $[0, 2\pi)$:
 $\sin 2x = \cos x$
- Use the Quadratic Formula to find all solutions in the interval $[0, 2\pi)$:
 $\tan^2 x - 6 \tan x + 4 = 0$